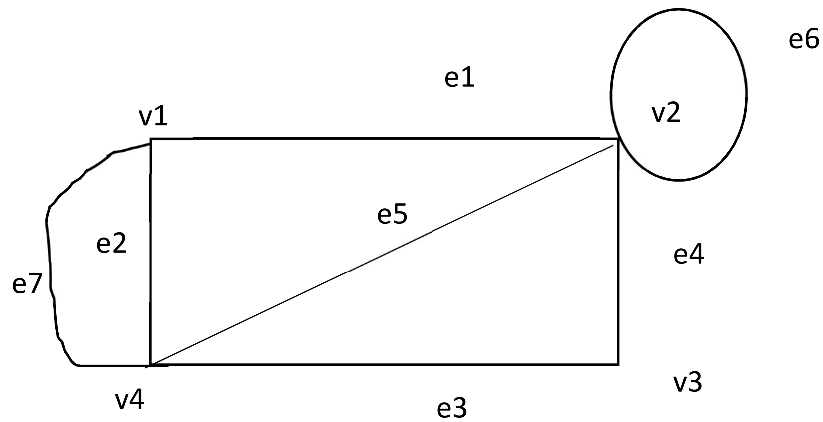
Graph Theory

Definition :-

Graph:- A linear† graph (or simply a graph) $G = (V, E)$ consists of a set of objects $V = \{v_1\ v_2, \ldots\}$ called vertices, and another set $E = \{e_1' \ e_2, \ldots\}$, whose elements are called edges, such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices.



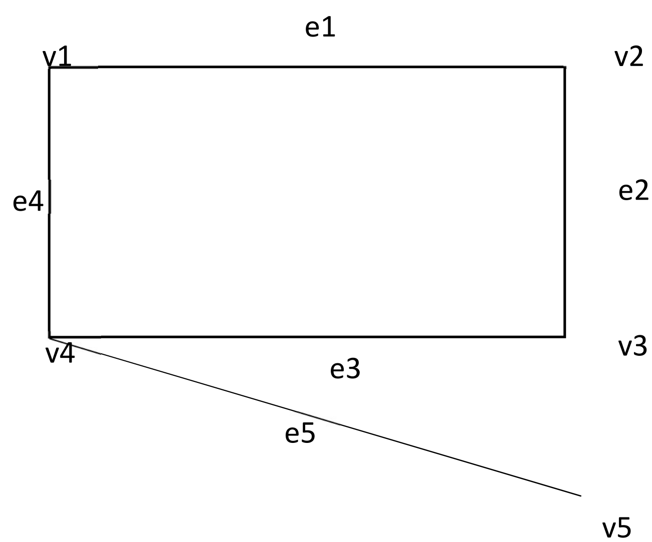
Graph with four vertices and seven edges.

Self-loop or Loop :- An edge having the same vertex as both its end vertices is called a self-loop (or simply a loop. for ex. Edge e6 in Fig. is a self-loop.

Parallel edges :- A more than one edge associated with a given pair of vertices, such edges is called parallel edges. for example, edges e2 and e7 in Fig.

Types of graph:-

1. simple graph:- A graph that has neither self-loops nor parallel edges is called a simple graph.



2. Null Graph-

- A graph whose edge set is empty is called as a null graph.
- In other words, a null graph does not contain any edges in it.

v1 * * v3

* v2

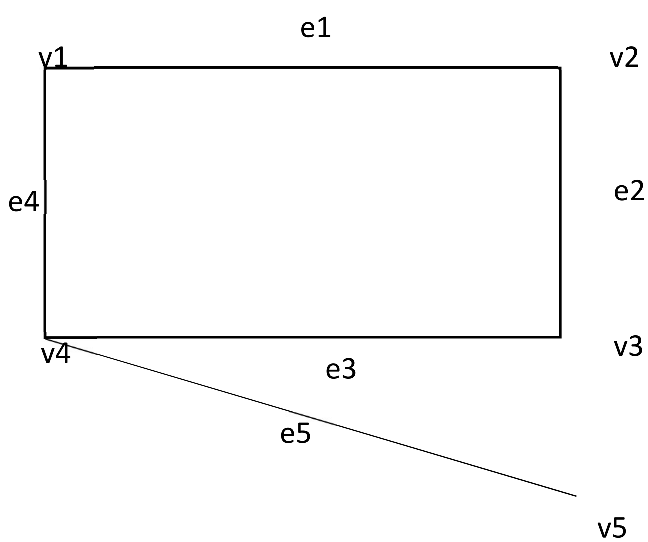
Here,

- This graph consists only of the vertices and there are no edges in it.
- Since the edge set is empty, therefore it is a null graph.

3. Non-Directed Graph-

- A graph in which all the edges are undirected is called as a non-directed graph.
- In other words, edges of an undirected graph do not contain any direction.

Example-



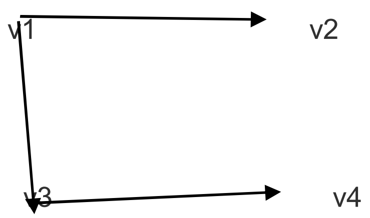
Here,

- This graph consists of four vertices and four undirected edges.
- Since all the edges are undirected, therefore it is a non-directed graph.

4. Directed Graph-

- A graph in which all the edges are directed is called as a directed graph.
- In other words, all the edges of a directed graph contain some direction.
- Directed graphs are also called as **digraphs**.

Example-



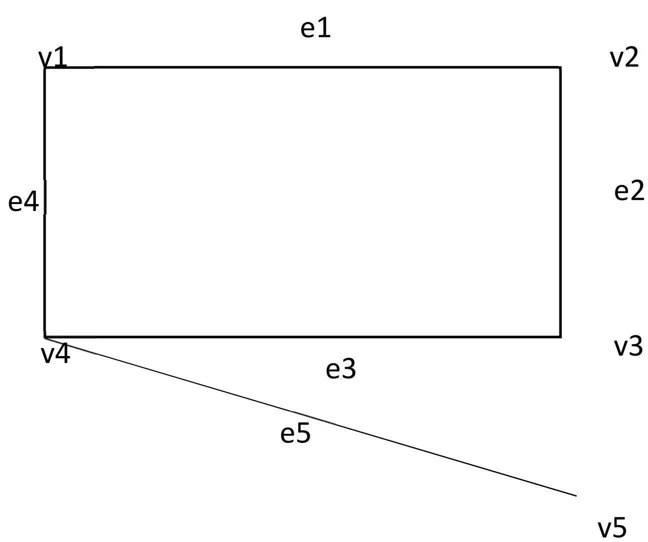
Here,

- This graph consists of four vertices and four directed edges.
- Since all the edges are directed, therefore it is a directed graph.

5. Connected Graph-

- A graph in which we can visit from any one vertex to any other vertex is called as a connected graph.
- In connected graph, at least one path exists between every pair of vertices.

Example-



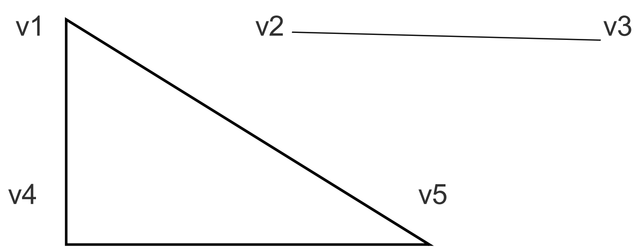
Here,

- In this graph, we can visit from any one vertex to any other vertex.
- There exists at least one path between every pair of vertices.
- Therefore, it is a connected graph.

6. Disconnected Graph-

- A graph in which there does not exist any path between at least one pair of vertices is called as a disconnected graph.

Example-



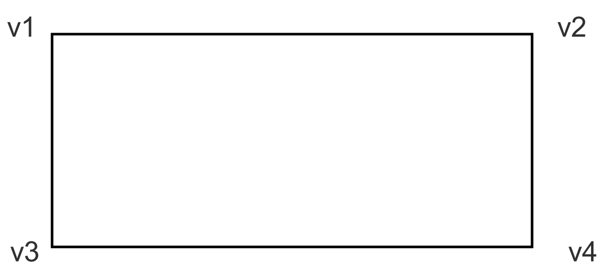
Here,

- This graph consists of two independent components which are disconnected.
- It is not possible to visit from the vertices of one component to the vertices of other component.
- Therefore, it is a disconnected graph.

7. Regular Graph-

- A graph in which degree of all the vertices is same is called as a regular graph.
- If all the vertices in a graph are of degree ‘k’, then it is called as a “**k-regular graph**“.

Examples-



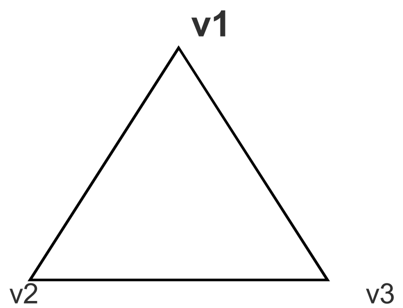
In these graphs,

- All the vertices have degree-2.
- Therefore, they are 2-Regular graphs.

8. Complete Graph-

- A graph in which exactly one edge is present between every pair of vertices is called as a complete graph.
- A complete graph of ‘n’ vertices contains exactly ${}^{n}C_2$ edges.
- A complete graph of ‘n’ vertices is represented as $\mathbf{K_n}$.

Examples-



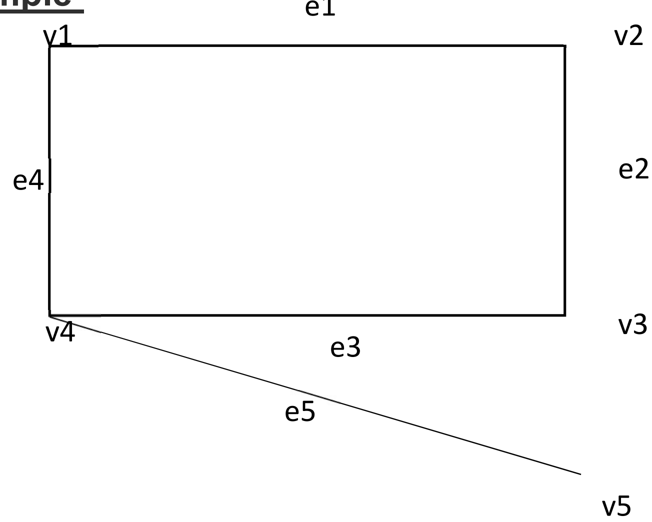
In these graphs,

- Each vertex is connected with all the remaining vertices through exactly one edge.
- Therefore, they are complete graphs.

9. Finite Graph-

- A graph consisting of finite number of vertices and edges is called as a finite graph.

Example-



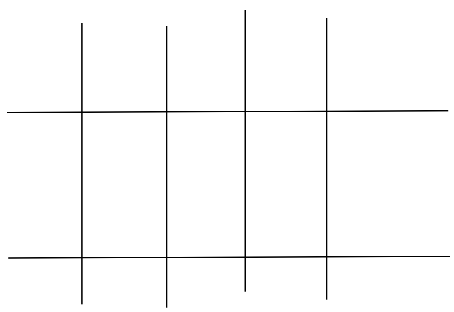
Here,

- This graph consists of finite number of vertices and edges.
- Therefore, it is a finite graph.

10. Infinite Graph-

- A graph consisting of infinite number of vertices and edges is called as an infinite graph.

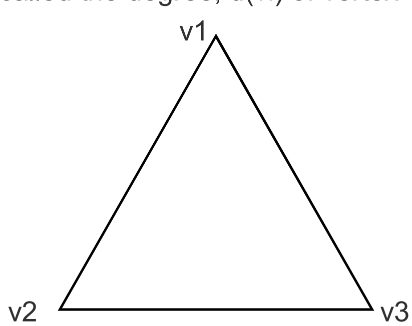
<u>Example-</u>



Here,

- This graph consists of infinite number of vertices and edges.
- Therefore, it is an infinite graph.

Degree Of Vertex :- The number of edges incident on a vertex vi, with self-loops counted twice, is called the degree, d(vi) of vertex vi



d(v1) = 2
d(v2) = 2
d(v3) = 2

The number of vertices of odd degree in a graph is always even.

Isolated vertex:- A vertex having no incident edge is called an isolated vertex.
In other words, isolated vertices are vertices with zero degree.

for ex. every vertex in a null graph is an isolated vertex.

Pendant vertex:- A vertex of degree one is called a pendant vertex.

for ex.

v1 ———————————— v2

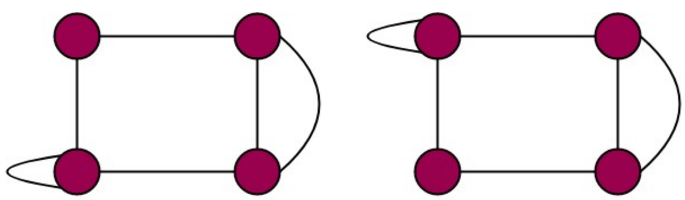
v1 and v2 are pendant vertex

ISOMORPHISM:- Definition Two graphs G and G′ are said to be isomorphic (to each other) if there is a one-to-one correspondence between their vertices and between their edges such that the incidence relationship is preserved**.**

by the definition of isomorphism that two isomorphic graphs must have
1. The same numbe of vertices.
2. The same number of edges.
3. An equal numberof vertices with a given degree.

Example:-

Show that the following two graphs isomorphic?



G1 G2

Solution-

Checking Necessary Conditions-

Condition-01:

Number of vertices in graph G1 = 4
Number of vertices in graph G2 = 4
Number of vertices in graph G1 = 4 = Number of vertices in graph G2

Here,

Both the graphs G1 and G2 have same number of vertices.
So, Condition-01 satisfies.

Condition-02:
Number of edges in graph G1 = 5

Number of edges in graph G2 = 5

Here,

Both the graphs G1 and G2 have different number of edges.
So, Condition-02 satisfies.

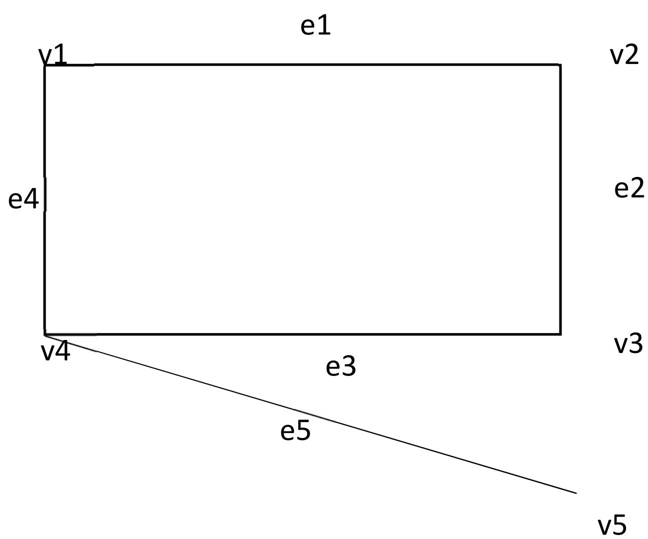
so given graphs can be isomorphic.

∴ G1 and G2 are isomorphic graphs.

Walks , Path, Circuits:-

A walk is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.
No edge appears (is covered or traversed) more than once in a walk.
A vertex, however, may appear more than once.

- The total number of edges covered in a walk is called as **Length of the Walk**.



v1 e1 v2 e2 v3 e3 v4 is a walk.

Terminal Vertices :-Vertices with which a walk begins and ends are called its terminal vertices.

Types of walks:-

There are two types of walks
1. open walk
2. closed walk

Open walk :- The vertices at which the walk starts and ends are different is called open walk.
for ex v1 e1 v2 e2 v3 e3 v4 is a open walk.

In graph theory, a walk is called as an **Open walk** if-

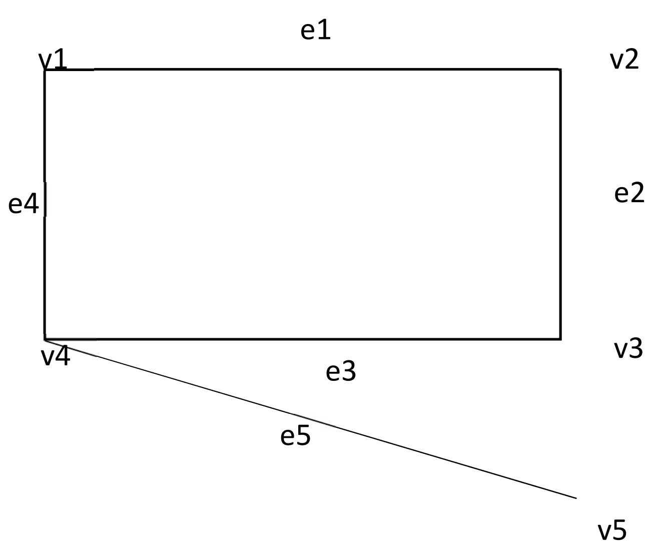
- Length of the walk is greater than zero
- And the vertices at which the walk starts and ends are different.

Closed walk:- The vertices at which the walk starts and ends are same is called closed walk.
v1 e1 v2 e2 v3 e3 v4 e4 v1 is a closed walk.

In graph theory, a walk is called as a **Closed walk** if-

- Length of the walk is greater than zero
- And the vertices at which the walk starts and ends are same.

Path:- An open walk in which no vertex appears more than once is called a path.



from fig. v1 e4 v4 e5 v5 is path. because starting and ending vertices are different, No edge appears (is covered or traversed) more than once in a walk.

Circuit :- A closed walk in which no vertex (except the initial and the final vertex) appears more than once is called a circuit.

from above fig. v1 e1 v2 e2 v3 e3 v4 e4 v1 is a circuit because it starting and ending vertex is same.

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