R
S
M
L

Definitions - Matrix

- A matrix is an array of numbers

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$$

- Denoted with a **bold Capital letter**
- All matrices have an order (or dimension):

that is, the number of rows × the number of columns. So, **A** is 2 by 3 or (2 × 3).

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Matrix Operations

- Addition
- Subtraction
- Scalar Multiplication
- Multiplication

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Addition of Matrices

- Two matrices may be added iff they are the same order.

- Simply add the corresponding elements.

  So, **A** + **B** = **C** yields

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Addition (cont.)

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

- Where

$$a_{11} + b_{11} = c_{11}$$
$$a_{12} + b_{12} = c_{12}$$
$$a_{21} + b_{21} = c_{21}$$
$$a_{22} + b_{22} = c_{22}$$
$$a_{31} + b_{31} = c_{31}$$
$$a_{32} + b_{32} = c_{32}$$

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Addition (cont.)

For example:- If $A = \begin{bmatrix} 7 & 5 \\ 6 & 9 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 0 & 6 \\ 4 & 7 \end{bmatrix}$ then find A+B

Solution:- The given Matrix $A = \begin{bmatrix} 7 & 5 \\ 6 & 9 \\ 5 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 0 & 6 \\ 4 & 7 \end{bmatrix}$

$$A+B = \begin{bmatrix} 5 & 2 \\ 6 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 0 & 6 \\ 4 & 7 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 5+2 & 2+3 \\ 6+0 & 3+6 \\ 1+4 & 0+7 \end{bmatrix} = \begin{bmatrix} 7 & 5 \\ 6 & 9 \\ 5 & 7 \end{bmatrix}$$

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Subtraction of Matrices

- Two matrices may be subtracted if and only if they are the same order.

- Simply subtract the corresponding elements.

  So, **A - B = C** yields

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Subtraction (cont.)

For example:- If A $=\begin{bmatrix} 7 & 5 \\ 6 & 9 \\ 5 & 7 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 3 \\ 0 & 6 \\ 4 & 7 \end{bmatrix}$ then find A-B

Solution:- The given Matrix $A = \begin{bmatrix} 7 & 5 \\ 6 & 9 \\ 5 & 7 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 3 \\ 0 & 6 \\ 4 & 7 \end{bmatrix}$

$$A\text{-}B = \begin{bmatrix} 5 & 2 \\ 6 & 3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 6 \\ 4 & 7 \end{bmatrix}$$

$$A\text{-}B = \begin{bmatrix} 5-2 & 2-3 \\ 6-0 & 3-6 \\ 1-4 & 0-7 \end{bmatrix} \qquad A\text{-}B = \begin{bmatrix} 3 & -1 \\ 6 & -3 \\ -3 & -7 \end{bmatrix}$$

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Scalar Multiplication:-

- To multiply a scalar times a matrix, simply multiply each element of the matrix by the scalar quantity

$$k\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix}$$

For example:- If A= $\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$ and k = 2 then find 2A

$$2A = 2\begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 10 \\ 8 & 12 \end{bmatrix}$$

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Multiplication of Matrix

- To multiply a matrix times a matrix, we write

  AB (A times **B)**

- In order to multiply matrices, they must be **Conformable** that is, the number of columns in A must equal the number of rows in B
- So,

  $\mathbf{A} \quad \times \quad \mathbf{B} \quad = \quad \mathbf{C}$

  $(m \times \mathbf{n}) \times (\mathbf{n} \times p) = (m \times p)$

- $(m \times n) \times (p \times n)$ = cannot be done
- $(1 \times n) \times (n \times 1)$ = a scalar (1x1)

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- AB does not necessarily equal BA
- (BA may even be an impossible operation)
- For example,

$$\begin{array}{ccccc} \mathbf{A} & \times & \mathbf{B} & = & \mathbf{C} \\ (2\times 3) & \times & (3\times 2) & = & (2\times 2) \\ \mathbf{B} & \times & \mathbf{A} & = & \mathbf{D} \\ (3\times 2) & \times & (2\times 3) & = & (3\times 3) \end{array}$$

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Matrix Multiplication (cont.)

- Thus

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} x \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix}$$

- Where

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31}$$
$$c_{12} = a_{11}b_{12} + a_{12}b_{22} + a_{13}b_{32}$$
$$c_{21} = a_{21}b_{11} + a_{22}b_{21} + a_{23}b_{31}$$
$$c_{22} = a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32}$$
$$c_{31} = a_{31}b_{11} + a_{32}b_{21} + a_{33}b_{31}$$
$$c_{32} = a_{31}b_{12} + a_{32}b_{22} + a_{33}b_{32}$$

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Matrix Multiplication:- an example

- Thus

$$\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix} x \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \\ c_{31} & c_{32} \end{bmatrix} = \begin{bmatrix} 30 & 66 \\ 36 & 81 \\ 42 & 96 \end{bmatrix}$$

- where

$$c_{11} = 1*1 + 4*2 + 7*3 = 30$$
$$c_{12} = 1*4 + 4*5 + 7*6 = 66$$
$$c_{21} = 2*1 + 5*2 + 8*3 = 36$$
$$c_{22} = 2*4 + 5*5 + 8*6 = 81$$
$$c_{31} = 3*1 + 6*2 + 9*3 = 42$$
$$c_{32} = 3*4 + 6*5 + 9*6 = 96$$