Rajarshi Shahu Mahavidyalaya(Autonomous),Latur

Department of Mathematics

Year: 2020-21

Syllabus for

B.Sc.-II (Mathematics)

CBCS Pattern

w.e.f. 2020-2021

Rajarshi Shahu Mahavidyalaya(Autonomous) , Latur

BoS in Mathematics

1. **Introduction:**

The courses for the UG Programme are framed using time tested and internationally popular text books so that the courses are at par with the courses offered by any other reputed universities around the world.

Only those concepts that can be introduced at the UG level are selected and instead of cramming the course with too many ideas the stress is given in doing the selected concepts rigorously. The idea is to make learning mathematics meaningful and an enjoyable activity rather than acquiring manipulative skills and reducing the whole thing an exercise in using thumb rules.

As learning Mathematics is doing Mathematics, to this end, some activities are prescribed to increase student's participation in learning. Duration of the degree programme shall be six semesters distributed in a period of three academic years.

2. Title of the Course: **B.Sc (Mathematics)**

3. Objectives of the Course:

Successful Mathematics students of this institute will gain lifelong skills, including following:

- To develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment.
- The development of their mathematical knowledge.
- Confidence by developing a feel for numbers, patterns and relationships.
- An ability to consider and solve problems and present and interpret results.
- Communication and reason using mathematical concepts.
- To develop an understanding of mathematical principles.
- To develop the abilities to reason logically, to classify, to generalize and to prove.
- To acquire a foundation appropriate to their further study of mathematics and of other disciplines.

4. Advantages of the Course:

Student will be getting highly motivated for higher studies in reputed institutes like IIT's , NIIT's,IISc's,CMI,HRI etc...

5. Duration of the Course:	Three years
6. Eligibility of the Course:	**B.Sc. I (One optional as Mathematics)**
7. Strength of the Students:	40
8. Fees for Course:	As per UGC/University/College rules.
9. Period of the Course:	As per UGC/University/College rules
10. Admission / Selection procedure:	As per UGC/University/College rules
11. Teacher's qualifications:	As per UGC/University/College rules
12. Standard of Passing:	As per UGC/University/College rules
13. Nature of question paper with scheme of marking:	As per UGC/University/College rules
15**. List of book recommended**:	Included in syllabus
16. **List of Laboratory Equipments, Instruments, Measurements etc.:**	Matlab Software with one computer Lab
17. **Rules and regulations and ordinance if any:**	As per UGC/University/College rules
18. Medium of the language:	English
19. Structure of the Course:	Attached as Annexure 'A'
20. Allotment of workload (Theory/Practical):	Attached as Annexure 'A'
21. Staffing pattern:	As per UGC/University/College rules.
22. Intake capacity of students:	As per UGC/University/College rules
23. Paper duration:	Each theory paper is of 45Contact hours
24. To be introduced from:	**B. Sc. II (Revised From June 2020)**

Chairman Board of Studies

Mathematics

(Mr. M. S. Waware)

List of BoS Member(2019-2022)

1. Dr. D D Pawar (VC Nominee)
   Director,
   School of Mathematical Sciences
   Swami Ramanand Teerth Marathwada University,
   Nanded.
2. Dr. S D Kendre (Subject Expert)
   Department of Mathematics,
   SPPU,Pune
3. Dr. M T Gophane (Subject Expert)
   Department of Mathematics
   Shivaji University ,Kolhapur .
4. Dr. A A Yadav
   R S M , Latur
5. Prof .S M Shinde (Student Alumni)
   Govt.College of Engg., Karad Dist :Satara
6. Mr. S S Ranmal
   Sungrace Computers Pvt Ltd, Pune
7. Prof. N . S. Pimple
   R S M , Latur
8. Dr. S B Birajdar
   R S M , Latur
9. Mr. S D Malegaokar
   R S M , Latur
10. Miss A B Kale
    R S M , Latur
11. Mr. D M Ghuge
    R S M , Latur

Program Outcomes:

On Successful UG Mathematics students of this institute will gain lifelong skills, including following:

- To develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment.
- Confidence by developing a feel for numbers, patterns and relationships.
- An ability to consider and solve problems and present and interpret results.
- Communication and reason using mathematical concepts.
- To understanding of mathematical principles and their applications
- To develop the abilities to reason logically, to classify, to generalize and to prove.
- To acquire a foundation appropriate to their further study of mathematics and of other disciplines.
- They Can qualify IIT-JAM entrance in the subject of Mathematics
- They will get admitted to ranked institute in India
- They can do minor research in the field of Maths
- They can solve problems independently

Annexure 'A'

Rajarshi Shahu Mahavidyalaya, Latur (Autonomous)

Department of Mathematics

B.Sc. II (Mathematics) Semester III

Curriculum Structure with effect from June, 2020

Code No.	Title of the course with paper number	Hours/ Week	Marks (50)		Credits
			In Sem	End Sem	
U-MAT-339	Real Analysis	03	20	30	02
U-MAT-340	Group Theory	03	20	30	02
U-MAT-341	Laboratory Course on **Problems in Real Analysis**	03	20	30	01
U-MAT-342	Laboratory Course on **Problems in Group Theory**	03	20	30	01
SEC-I U-ADC-334-R	Skill Enhancement Course –I **R software**	01Theory +2 Practical	20	30	02
	Total Credits	15 Lecture			08

Student Stay Hours: 15/Week

B.Sc.II (Mathematics) Semester IV

Code No.	Title of the course with paper number	Hours/ Week	Marks (50)		Credits
			In Sem	End Sem	
U-MAT-439	Ordinary Differential Equations	03	20	30	02
U-MAT-440	Ring Theory	03	20	30	02
U-MAT-441	Laboratory Course on **Problems in Ordinary Differential Equations**	03	20	30	01
U-MAT-442	Laboratory Course on **Problems in Ring Theory**	03	20	30	01
SEC-II U-ADC-434-R	Skill Enhancement Course -II **R software**	01Theory +2 Practical	20	30	02
	Total Credits	15 Lectures			08

Student Stay Hours: 12/Week

B. Sc. –II [Mathematics] Semester III

Course Code: U-MAT-339

Paper-V Real Analysis

Credits:02 **Marks : 50** **Total Hours : 45**

Learning Objectives :

- Definition of Sequence and it's properties
- Bolzano-Weierstrass theorem and Cauchy's criterion for convergence
- Sequence of functions
- Infinite Series and Absolute Convergence

Course Outcomes:

On completion of this unit successful students will be able to:

- Know the definition of the limit of a sequence, evaluate the limits of a wide class of real sequences;
- Do point wise and Uniform convergence
- Decide on convergence or divergence of a wide class of series;
- determine whether or not real series are convergent by comparison with standard series or using the Ratio Test;
- understand the concept of Absolute convergence and be familiar with the statements and some proofs of the standard results about Absolute convergence;

Unit I : Sequences [15 lectures]

Sequences and their limits, limit theorems,Monotone Sequences, Subsequences and Bolzano-Weierstrass theorem, The Cauchy's criterion, Properly divergent sequences

UNIT II : Sequence of functions

Pointwise and uniform convergence,Interchange of limits ,The exponential and Logarithmic functions, The trigonometric functions [15 lectures]

UNIT III : Infinite Series

Introduction to series, Cauchy's criterion for series, Comparison tests, Absolute convergence Test for Absolute convergence, Test for Non- absolute convergence, series of functions. [15 lectures]

References :

1. Introduction to real Analysis (Third Edition) Robert G.Bartle & Donald R Sherbert (Wiley Student Edition)
2. Calculus and Analytical Geometry (Sixth Edition) By George B. Thomas, Ross L. Finney (Narosa Publishing House)
3. Advanced Calculus (Third Edition) By Robert Wrede & Murray R. Spiegel
4. Mathematical Analysis ,By S.C.Malik & Savita Arora
5. Methods of Real Analysis ,By R. Goldberg
6. Elements of Real Analysis,By Shanti Narayan M. D. Raisinghaniya (S.Chand & Comp. Ltd.)

Course Code: U-MAT-340

Paper-VI Group Theory

Credits:02 **Marks : 50** **Total Hours : 45**

Learning Objectives:

- **Definition and examples of groups**
- **Subgroups, Normal Subgroup, Quotient Group, Homomorphism, Isomorphism**
- **Cayley and Lagrange theorem**

Course Outcomes:

Students will able to

- **Categorize group structures**
- **Find quotient group, subgroups of a given group**
- **Analyze group structure from its order**

Unit-I : Groups and Subgroup

Definition of group,subgroups, Elementry properties of groups, finite groups, cyclic groups and its properties. **[15 Lectures]**

Unit- II Permutation groups and isomorphism

Symmetric groups, Permutations, Group isomorphism, Automorphism and their properties, Cayleys theorem, **[15 Lectures]**

Unit-III Coset and Lagrange's theorem **[15 Lectures]**

Definition of coset and properties, Lagrange's theorem and its consequences, an applications of cosets to permutation groups. External direct product, definition and examples of normal subgroups and factor groups.

Reference Books

1. **Contemporary Abstract Algebra**, By Joseph Gallion, Narosa Publications.
2. **A first course in abstract algebra**, By J.B. Fraleigh, Narosa Publications.
3. **Topic in Algebra**, By I.N. Herstein (Second Edition)
4. **Modern Algebra**, By A.R. Vasishtha, Krishna Prakashan Media.
5. **Modern Algebra**, By R.P. Rohtatgi, Dominant Publishers and Distributors, New Delhi.
6. **Modern Algebra**, By Goyal and Gupta, Pragato Prakashan Meerut
7. **College Mathematics**, By N.R. Jayaram and R.V. Prabhakara, Himalaya Publishing House.
8. **Lectures on Abstract Agebra By T M Karade** J N Salunke ,K S Adhav, Maya Bendre, Sonu- Nilu Publication
9. **First Course in Abstract Algebra**, By Vijay K.Khanna,and M L Bhambri Vikas Publication Company
10 **Topics in Abstract Algebra** (second Edition) by M K Sen, Shamik Ghosh,Parthasarathi Mukhopadhyay,Universities Press(India)

Course Code: U-MAT-341
Laboratory Course-III
Problems in Real Analysis

Learning Objectives:

- Convergence and divergence of sequences
- Some special sequence of functions
- Infinite series

Course Outcomes:

After successful completion of the course students are able

- Solve example on convergence and divergence of sequences
- Handle the problems on pointwise and uniform convergence
- Discus the convergence of infinite series

List of Practical's

1. Using the definition of the limit of a sequence to establish the following limits.

   a) $\lim\left(\frac{n}{n^2+1}\right)=0$

   b) $\lim\left(\frac{3n+1}{2n+5}\right)=\frac{3}{2}$

   c) $\lim\left(\frac{2n}{n+1}\right)=2$

   d) $\lim\left(\frac{n^2-1}{2n^2+3}\right)=\frac{1}{2}$

2. Show that :

   a) $\lim\left(\frac{1}{\sqrt{n+7}}\right)=0$ b) $\lim\left(\frac{\sqrt{n}}{n+1}\right)=0$ c) $\lim\left(\frac{2n}{n+2}\right)=2$

3. Show that :

   a) $\lim\left(\frac{1}{n}-\frac{1}{n+1}\right)=0$ b) $\lim\left(\frac{1}{3^n}\right)=0$ c) $\lim\left((2n)^{1/n}\right)=1$

4. Show that: $\lim\left(\frac{n^2}{n!}\right)=0$

5. Show that if X and Y are sequences such that X and X+Y are convergent, then prove that Y is convergent.

6. Show that the following sequences are not convergent :

   a) $\left(2^n\right)$ b) $\left((-1)^n n^2\right)$

7. If 0<a<b, determine $\lim\left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right)$

8. If a>0,b>0 show that $\lim\left(\sqrt{(n+a)(n+b)}-n\right)=(a+b)/2$

9. Let $x_1 = 8$ *and* $x_{n+1} = \frac{1}{2} x_n + 2$ *for* $n \in N$ .Show that (x_n) is bounded and monotone. Find the limit.

10. Let $x_1 = 1$ *and* $x_{n+1} = \sqrt{2 + x_n}$ *for* $n \in N$ .Show that (x_n) converges and find the limit.

11.Investigate the convergence or divergence of the following sequences:

a) $\sqrt{n^2 + 2}$ b) $(\sqrt{n}/(n^2+1))$ c) $(\sqrt{n^2+1}/\sqrt{n})$ d) $(\sin\sqrt{n})$.

12. Show that $\lim\left(\frac{x}{x+n}\right) = 0,\ for\ all\ x\ in\ R,\ x \geq 0.$

13. Show that $\lim\left(\frac{nx}{1+nx}\right) = 0,\ for\ all\ x\ in\ R,\ x \geq 0.$

14. Show that $\lim\left(\frac{x^n}{1+x^n}\right) = 0,\ for\ all\ x\ in\ R,\ x \geq 0.$

15.Let $f : \Re \to \Re$ be uniformly continuous on $\Re$ and let $f_n(x) = f(x + 1/n)$ *for* $x \in \Re$ .Show that (f_n) converges uniformly on $\Re$ to f .

16.The exponential function satisfies the following properties :

i) $E(x) \neq 0\ for\ all\ x \in R$ ii) $E(x+y) = E(x)E(y)\ \ for\ all\ x, y \in R;$

iii) $E(r) = e^r\ \ for\ \ all\ r \in Q$

17. Calculate e correct to 5 decimal places.

18. Show that $|\sin x| \leq 1\ and\ |\cos x| \leq 1\ for\ all\ \ x \in R$.

19. Calculate cos(.2),sin(.2) and cos 1,sin 1 correct to four decimal places.

20. Show that if $0 \leq x \leq a\ and\ n \in N$, then

$$1 + \frac{x}{1!} + ... + \frac{x^n}{n!} \leq e^x \leq 1 + \frac{x}{1!} + ... + \frac{x^{n-1}}{(n-1)!} + \frac{e^a x^n}{n!}\ .$$

21. Does the series $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n}}\right) convegre?$

22. Does the series $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+1}-\sqrt{n}}{n}\right) convegre\,?$

23. Discuss the convergence or the divergence of the series whose nth term is

i) $(n(n+1))^{-1/2}$ ii) $(n^2(n+1))^{-1/2}$

24. Show that the series $\frac{1}{1^2} + \frac{1}{2^3} + \frac{1}{3^2} + \frac{1}{4^3} + ...$ is convergent, but both the ratio and the Root Tests fail to apply.

25. Show that the series $1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + + - ...$ is divergent.

26. If a and b are positive numbers, then $\sum (an+b)^{-p}$ *converges if* $p > 1$ and diverges if $p \leq 1$.

27. Test the following series for convergence and for absolute convergence.

i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2+1}$ ii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$ iii) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n+2}$

28. Discuss the series whose nth term is:

i) $(-1)^n \frac{(n)^n}{(n+1)^{n+1}}$ ii) $\frac{(n)^n}{(n+1)^{n+1}}$

29. Determine the radius of convergence of the series $\sum a_n x^n$,where a_n is given by :

i) $1/n^n$ ii) $\frac{n^\alpha}{n!}$

30. If $\sum a_n$ is an absolutely convergent series, then the series $\sum a_n \sin nx$ is uniformly convergent, then $\lim(nc_n) = 0$

Course Code U-MAT-342
Laboratory Course-IV
Problems in Group Theory

Credits:01 **Marks : 50** **Total Hours : 45**

Learning Objectives:

- Finite and infinite group examples
- Examples on subgroups
- Examples on isomorphism

Course Outcomes:

After successful completion of the course students are able

- Solve example on group structure
- Handle the problems on cyclic groups
- Discus the isomorphism of two group structures

List of practical's

1. Prove that the set GL(2,R) = $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / a, b, c, d \in \mathbb{R}, ad - bc \neq 0\right\}$ of 2×2 matices with real entries and non-zero determinant is non abelian group under the operation of matrix multiplication?
2. Show that the set $\{5,15,25,35\}$ is a group under multiplication modulo 40. What is identity of this group? Can you see any relation between?
3. Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in GL(2, $\mathbb{Z}_{11}$)
4. Prove that a group G is abelian if and only if $(ab)^{-1}=a^{-1}b^{-1}$ for all a,b in G
5. Construct Cayley table for U(12)
6. Find all subgroups of D_3 and find how many subgroups of order does D_3?
7. In the group $\mathbb{Z}$ find
   i. <{8,4}>
   ii. <{8,15}>
   iii. <{6,15}>
   iv. <{m,n}>
   v. <{12,18,45}>
8. Let $\mathbb{R}^*$ be the group of nonzero real numbers under multiplication and let H=$\{x \in \mathbb{R}^* / x^2 \text{ is rational}\}$ prove that H is subgroup of $\mathbb{R}^*$. Can the exponent 2 be replaced by any positive integer and still have H be a group?
9. Find all generators of $\mathbb{Z}_6$, $\mathbb{Z}_8$, $\mathbb{Z}_{30}$.
10. Determine the number of cyclic groups of order 4 in D_n
11. How many elements of order 5 are their in A_6
12. Determine all possible order of elements of S_7.
13. Show that number of cyclic group of order n upto isomorphism is 1. Hence find number of groups of order p upto isomorphism for any prime p.

14. Let G be a group. Prove that the mapping $a(g)=g^{-1}$ $\forall$ $g\epsilon G$ is automorphism iff G is abelian.
15. Show that $\mathbb{Z}$ has infinitely many subgroup isomorphic to $\mathbb{Z}$
16. Show that the mapping from U(16) to itself given by $x\to x^3$ is an automorphism. What about $x\to x^5$ and $x\to x^7$? Generalixe.
17. Show that the group of all rational numbers under addition is not isomorphic to set group of all real numbers under addition.
18. Show that ($\mathbb{R}$+, .) is isomorphic to ($\mathbb{R}$,+)
19. Discuss the group of symmetries of an equilateral triangle
20. Discuss the group of symmetries of a square
21. Comment on converse of Lagrange's Theorem.
22. Show that order of proper subgroup of a group of order 75 can be at most 75
23. Let $|G| = 15$. If G has only one subgroup of order 3 and only one subgroup of order 5 then prove that G is cyclic group. Generalize to $|G| = pq$. Where p and q are prime
24. Let G be a group of order 25. Prove that G is cyclic or g^5=e for all g ϵ G. Generalize to any group of order p^2 where p is prime.
25. Determine all finite subgroups of $\mathbb{C}$*(Group of all nonzero complex numbers under multiplication).
26. Find the number of elements of order 5 in $\mathbb{Z}_{25}\oplus\mathbb{Z}_5$.
27. Prove or disprove that
    a. $\mathbb{Z}\oplus\mathbb{Z}$ is cyclic
    b. $\mathbb{Z}_2\oplus\mathbb{Z}_3$ is cyclic
    c. $\mathbb{Z}_4\oplus\mathbb{Z}_6$ is cyclic
28. Prove that $\mathbb{Z}_m\oplus\mathbb{Z}_n$ is cyclic if and only if gcd(m,n)=1.
29. Prove that every subgroup H of a Group G such that $[G:H] = 2$ is normal. hence $G/_H$.
30. Prove that id a group G has unique subgroup H of some finite order, then H is normal in G.

Skills Enhancement Course-I
R software U_ADC-334-R

Learning Objectives:

- Use of R Calculator
- Matrix operations

Course Outcomes:

On completion of this course successfully students will be getting following skills:

- Using loops in R software
- Sorting and ordering and lists in R software

Skill- I

Basic fundamentals, installation and use of software, data editing, use of R as a calculator, functions and assignments.Use of R as a calculator, functions and matrix operations, missing data and logical operators.

Skill-II

Conditional executions and loops, data management with sequences. Data management with repeats, sorting, ordering, and lists

Reference Books

1. Introduction to Statistics and Data Analysis - With Exercises, Solutions and Applications in R By Christian Heumann, Michael Schomaker and Shalabh, Springer, 2016

2. The R Software-Fundamentals of Programming and Statistical Analysis -Pierre Lafaye de Micheaux, Rémy Drouilhet, Benoit Liquet, Springer 2013

3. A Beginner's Guide to R (Use R) By Alain F. Zuur, Elena N. Ieno, Erik H.W.G. Meesters, Springer 2009

B. Sc. –II [Mathematics] Semester IV

Course Code: U-MAT-439
Paper-VII Ordinary Differential Equations

Credits:02 Marks : 50 Total Hours : 45

Learning Objectives:

- Concepts of ODE
- Linear equations with constant coefficients
- Rules for finding Complementary function and Particular integral
- Linear equations with variable coefficients

Course Outcomes:

On completion of this course successfully students will be able to:

- Obtain general solutions to first-order, second-order, and higher-order homogeneous and non-homogeneous differential equations by manual and technology-based methods.
- Identify and apply initial and boundary values to find particular solutions to first-order, second-order, and higher order homogeneous and non-homogeneous differential equations by manual and technology-based methods, and analyze and interpret the results.
- Select and apply appropriate methods to solve differential equations; these methods will include, but are not limited to, undetermined coefficients, variation of parameters.

Unit I: Definitions and Formation of Differential Equation [10 lectures]

Preliminaries: Ordinary and partial differential equations order and degree, Solutions and constants of integration, The derivation of differential equation. Solutions, general, particular, singular, Equations of the First order and of the First Degree.

Unit II: Linear Differential equations with constant coefficient [18 lectures]

Linear equations, The complementary function, the particular Integral, the complete integral. The linear equation with constant coefficient and second member zero. Case of the auxiliary equation having equal roots, imaginary roots, The symbol *D*, Theorem concerning *D*. The linear equation with constant coefficient and second member is a function of x, The symbolic function $\frac{1}{f(D)}$ Method of finding the Particular integral

Unit III: Linear Differential equations with variable coefficients [17 lectures]

The Homogeneous linear equation. First method of solution , Second method of Solution to find the Complementary function, particular integral. The symbolic functions $f(\Theta)$ and $\frac{1}{f(\theta)}$ .Method for finding particular integral. Integral corresponding to a term of form x^{α} in the second member, Equations reducible to the homogeneous linear form, Orthogonal Trajectories Equation of the second order ,Complete solution in terms of a known integral ,by inspection, by means of first two integrals, by variation of parameters. Ordinary differential equation with more than two variables .Simultaneous linear differential equation and their solutions. Geometrical meaning , method of finding the solution of the single integrals.

References :

1. Introductory Course in Differential Equations –By Daniel A. Murray (Orient Longman Limited)
2. M.D.Raisinghania," Ordinary and Partial Differential Equations" S.Chand and Company Ltd
3. G. F. Simmons, "Differential Equations with Applications and Historical Notes",Second Edition,Mc Graw Hill.
4. W. E. Williams, " Partial Differential Equations", Claredon Press Oxford.
5. G. Birkhoff and G. C. Rota, "Ordinary Differential Equations", John Wiley and Sons.
6. E. T. Copson, " Partial Differential Equations ", Cambridge University Press.
7. Differential Equation ,T M Karade Sonu –Nilu Publication

Course Code: U-MAT-440

Paper-VIII Ring Theory

Credits:02 **Marks : 50** **Total Hours : 45**

Learning Objectives

- Definition and some classes of Rings
- Ideals, Quotient ring.
- Euclidean rings and their properties.
- Reducibility and irreducibility of polynomial

Course Outcomes:

After successful completion of this course Students are able to

- Analyze classes of rings
- Find quotient structure of quotient ring .
- Learn Euclidean ring and examples .

Unit I:

Definition and examples of rings, some special classes of rings, Homeomorphisms, Isomorphism [15 Lectures]

Unit II:

Ideals and quotients rings, More ideals and quotients rings, the field of quotients of an integral domains . [15 Lectures]

Unit III:

Euclidean rings, A particular Euclidean ring (Ring of Gaussian Integers), Polynomial rings, Polynomial oven the rational fields. [15Lectures]

Reference Books:

1)Topics in Algebra, I.N. Herstein, John Wiley and Sons (New York)

2) A first course in abstract algebra, by J.B. Fraleigh, Narosa Publications.

3) Contemparary Abstract Algebra, by Joseph Gallion,Narosa Publications.

4) Modern Algebra, by A.R. Vasishtha, Krishna Prakashan Media.

5) Modern Algebra, by R.P. Rohtatgi, Dominant Publishers and Distributors, New Delhi.

6) Modern Algebra, By Goyal and Gupta, Pragato Prakashan Meerut

7)College Mathematics, by N.R. Jayaram and R.V. Prabhakara, Himalaya Publishing House.

8) Elements of Logic and Modern Algebra, by M.V. Bhat and M.L. Bhave, S. Chand and Company Ltd. Ramnagar, New Delhi 110055

9) Lectures on Abstract Agebra By T M Karade J N Salunke ,K S Adhav, Maya Bendre, Sonu-Nilu Publication

Course Code: U-MAT-441

Laboratory Course-V

Problem Course on Ordinary Differential Equations

Credits:01	Marks : 50	Total Hours : 45

Learning Objectives:

- Examples on Ordinary Differential Equations
- Examples on Linear equations with constant coefficients
- Examples on finding Complementary function and Particular integral
- Examples on Linear equations with variable coefficients

Course Outcomes:

On completion of this course successfully students will be able to:

- Solve Examples on general solutions to first-order, second-order, and higher-order homogeneous and non-homogeneous differential equations by manual and technology-based methods.
- Identify and apply initial and boundary values to find particular solutions to first-order, second-order, and higher order homogeneous and non-homogeneous differential equations by manual and technology-based methods, and analyze and interpret the results.
- Select and apply appropriate methods to solve differential equations; these methods will include, but are not limited to, undetermined coefficients, variation of parameters.

List of Practical's

1. Derive the differential equation $(x-a)^2+(y-b)^2=r^3$

2. Find the differential equation of all circles which pass through the origin and whose centre are on 9x-axis .

3. Solve 3ex tany dx + $(1-e^x)\sec^2 y\,dy=0$.

4. Solve $(4x+3x)\frac{dy}{dx}+y-2x=0$.

5. Solve $(2ax+by+g)dx+(2cy+bx+e)dy=0$.

6. Solve $(x^2y+2xy^2)dx-(x^3-3x^2y)dy=0$.

7. Solve $(y^4+2y)\,dx+(xy^3+2y^4-4x)\,dy=0$.

8. Solve $\cos^2 x\frac{dy}{dx}+y=\tan x$.

9. Solve $\frac{dy}{dx}+\frac{2}{x}$ y =3x^2 y$^{3|4}$.

10. Solve $\frac{d^3y}{dx^3}$ _3 $\frac{d^2y}{dx^2}$ +4y =0

11.$\frac{d^3y}{dx^3}$ +2$\frac{d^2y}{dx^2}$ +$\frac{dy}{dx}$ = e^{2x}+x^2+x .

12. Solve $\frac{d^2y}{dx^2}$ -$\frac{dy}{dx}$ +4y = e^x cosx+sin2x .

13.Find the orthogonal trajectories of the system of curves r^n sinn$\boldsymbol{\theta}$ = a^n

15.Using method of variation of parameter solve i) 6 y$''$ +5y' -6y =x . ii) y'' +y = 2sinx .sin2x .

16.Solve x$^2\frac{d^2y}{dx^2}$ – x $\frac{dy}{dx}$ + y= 2 logx **17.Solve** x$^2\frac{d^2y}{dx^2}$ +2x$\frac{dy}{dx}$ -20y=(x +1)^{2}.

17. Solve $(5+2x)^2\frac{d^2y}{dx^2}-6(5+2x)\frac{dy}{dx}+8y=0$

18. Show that the system of parabolas y^2= 4a(x+a) is self orthogonal .

19. Solve $(2x-1)^3\frac{d^3y}{dx^3}+(2x-1)\frac{dy}{dx}-2y=0$

20. Solve $x\ \frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}-y=e^x$

21. Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-y=0$ given that $x+\frac{1}{x}$ is an integral

22. Solve $3x^2\frac{d^2y}{dx^2}+(2x-6x^2)\frac{dy}{dx}-4y=0$

23. solve $\frac{dx}{dt}-7x+y=0,\qquad \frac{dy}{dt}-2x-5y=0$

24. Solve $\frac{dx}{x^2-y^2-z^2}=\frac{dy}{2xy}=\frac{dz}{2xz}$

25. Solve $(y+z)dz+dy+dz=0$

26. Solve $\frac{adx}{(b-c)xy}=\frac{bdy}{(c-a)zx}=\frac{cdz}{(a-b)xy}$

27. Solve $(y+z)dx+(2+x)dy+(x+y)dz=0$

28. solve $\frac{dx}{dt}+4x+3y=t,\qquad \frac{dy}{dt}+2x+5y=e^t$

29. Solve $(x-3)\ \frac{d^2y}{dx^2}-(4x-9)\frac{dy}{dx}+3(x-2)y=0,\ e^x$ being a solution

30. Verify that the function $\varphi_1$satisfies the equation, and find a second independent solution

$x^2y''-7xy'+15y=0,$ φ_1(x)=x^3.

U-MAT-442

Laboratory Course -VI

Problem Course on Ring Theory

Credits:01 **Marks : 50** **Total Hours : 45**

Learning Objectives

- Definition and some classes of Rings
- Ideals, Quotient ring.
- Euclidean rings and their properties.
- Reducibility and irreducibility of polynomial

Course Outcomes:

After successful completion of this course Students are able to

- Solve examples on rings
- Find quotient structure of quotient ring .
- Learn Euclidean ring and examples on it .

1. If R is Ring and $a, b, c, d \in R, then$ evaluate $(a + b)(c + d)$.
2. Prove that if R is Ring and $a, b \in R,$ then $(a + b)^2 = a^2 + ab + ba + b^2$, where by x^2 we mean xx
3. Find the form of the Binomial theorem is a general ring; in other words, find an expression for $(a + b)^n$, where n is a positive integer.
4. If every if $x \in R,$ satisfies x^2=x, prove that R must be commutative (A ring in which x^2 = x for all elements is called a Boolean Ring.)
5. If R is a ring, merely considering it as an abelian group under its addition, what is meant by na, where $a \in R$ and n is an integer. Prove that if $a, b \in R,$ and n, m are integers then $(na)(mb) = (nm)(ab).$
6. If D is an integral domain and D is of finite Characteristics, prove that the characteristics of D is a prime number.
7. Give an example of an integral domain which has an infinite number of elements yet is of finite Characteristics.
8. If D is an integral Domain and if na=0 for some $a \neq 0$ in D and some integer $n \neq 0$, prove that D is of finite characteristics.
9. If R is system satisfying all the conditions for a ring with unit element with the possible exception of $a + b = b + a$, prove that the axiom $a + b = b + a$ must hold in R is thus a Ring.
10. Show that the commutative ring D is an integral domain iff for $a, b, c, \in Da \neq 0$ the relation $ab = ac$ implies that $b = c$.

11. Prove that the any field is an integral Domain.
12. Let $J(\sqrt{2})$ be all real number of the form $m + n\sqrt{2}$, where m, n are integers; Prove that $J(\sqrt{2})$ forms a ring under the usual addition and multiplication of real numbers. Define $\emptyset: J(\sqrt{2}) \to J(\sqrt{2})$ by $\emptyset(m + n\sqrt{2}) = m - n\sqrt{2}$.Verify that $\emptyset$ is Homomorphism of $J(\sqrt{2})$onto $J(\sqrt{2})$and its kernel $I(\emptyset)$, consists only of 0.
13. Let J be the ring of integers, J_n the ring of integers modulo n. Define $\emptyset: J \to J_n$by $\emptyset(a)$= reminder of a on division by n then verify that $\emptyset$ is Homomorphism of J onto $J_n$and that the kernel $I(\emptyset)$, consists of all multiples of n.
14. If U is an ideal of R and $1 \in U$, prove that U=R.
15. If F is a field, prove that's its only ideal is (0) and f itself.
16. Prove that any homomorphism of field is either an isomorphism or takes each element into 0.
17. If R is Commutative ring and $a, \epsilon R$,
    a. Show that $aR = \{ar | r \in R\}$ is a two sided ideal of R
    b. Show by an example that this may be false if R is not Commutative.
18. If U, V are ideals of R, let $U + V = \{u + v | u \in U, v \in V\}$. Prove that $U + V$ is also an ideal.
19. If U is an ideal of R, let $r(U) = \{x \in R | xu = 0 \ for\ all\ u \in U\}$ prove that r(U) is an ideal of R.
20. Let R be a ring with unit element. Using its element, we define ring R by defining $a \oplus b = a + b + 1$, and $a.b = ab + a + b$ right hand side of these relations are those of R.
    a. Prove that R is Ring under the operation $\oplus$ and
    b. What acts as the zero-element of R?
    c. What acts as the unit-element of R?
    d. Prove that R is isomorphic to R.
21. Let R be a Ring with unit element, R not necessarily commutative, such that the only right-ideals of R are (0) and R. prove that R is a Division Ring.
22. Let J be a ring of integers, p a prime number, and (p) the ideal of J consisting of all multiples of p. prove
    a. J|(p) is isomorphic to J_p, the ring of integer modulo p
    b. J_p is an field.
23. Let R be the ring of all real-valued continuous functions on the closed unit interval. If M is a Maximal ideal of R, prove that there exists a real number γ, $0 \leq \gamma \leq 1$, such that $M = M_\gamma = \{f(x) \in R | f(\gamma) = 0\}$.
24. Prove that if $[a, b] = [a', b']$ and $[c, d] = [c', d']$ then $[a, b][c, d] = [a', b'][c', d']$
25. Prove that Distributive law in F
26. Prove that the mapping $\emptyset: D \to F$ defined by $\emptyset(a) = [a, 1]$ is an isomorphism of D into F.
27. Find all the units in $J[i]$,F[X]
28. If $a + bi$ is not a unit of $J[i]$ prove that $a^2 + b^2 > 1$.
29. Find the greatest common divisor in $J[i]\ of$
    a. $3 + 4i\ and\ 3 - 4i$
    b. $11 + 7i\ and\ 18 - i$

30. Find the Greatest common divisor of the following polynomials over F, the field of rational numbers:
    a. $x^3 - 6x^2 + x + 4$ and $x^5 - 6x + 1$
    b. $x^2 + 1$ and $x^6 + x^3 + x + 1$
31. Prove that:
    a. $x^2 + x + 1$ is irreducible over F, the field of integers mod 2.
    b. $x^2 + 1$ is irreducible over F, the integers mod 7.
    c. $x^3 - 9$ is irreducible over F, the integers mod 91.
    d. $x^3 - 9$ is irreducible over the integers mod 11.
32. Prove that $x^2 + 1$ is irreducible over the field F of integers mod 11 and prove directly that $F[x]/(x^2 + 1)$ is a field having 121 elements.
33. Prove that $x^2 + x + 4$ is irreducible over F, the field of integers mod 11.and prove directly that
    $F[x]/(x^2 + x + 4)$ is a field having 121 elements.

34. Let F be the field of real numbers. Prove that $F[x]/(x^2 + 1)$ is a field isomorphic to the field of complex numbers.
35. If p is prime number, prove that the polynomial $x^n - p$ is irreducible over the rationals.
36. If a is rational and $x - a$ divides an integers monic polynomial, prove that the a must be an integer.

Skills Enhancement Course-II

R software –II U-ADC-434-R

Credits:02 **Marks : 50** **Total Hours : 45**

Learning Objectives:

- Data Management
- Statistical functions

Course Outcomes:

On completion of this course successfully students will be getting following skills:

- Importing of external data in various file formats
- Handling of data through graphics

Skill-III

Vector indexing, factors, Data management with strings, display and formatting. Data management with display paste, split, find and replacement, manipulations with alphabets, evaluation of strings, data frames. Data frames, import of external data in various file formats, statistical functions, compilation of data.

Skill-IV

Graphics and plots, statistical functions for central tendency, variation, skewness and kurtosis, handling of bivarite data through graphics, correlations, programming and illustration with examples.

Reference Books

1. Introduction to Statistics and Data Analysis - With Exercises, Solutions and Applications in R By Christian Heumann, Michael Schomaker and Shalabh, Springer, 2016

2. The R Software-Fundamentals of Programming and Statistical Analysis -Pierre Lafaye de Micheaux, Rémy Drouilhet, Benoit Liquet, Springer 2013

3. A Beginner's Guide to R (Use R) By Alain F. Zuur, Elena N. Ieno, Erik H.W.G. Meesters, Springer 2009