# Rajarshi Shahu Mahavidyalaya, Latur

# (Autonomous)

# BoS in Mathematics

1. **Introduction:**

The courses for the UG Programme are framed using time tested and internationally popular text books so that the courses are at par with the courses offered by any other reputed universities around the world.

Only those concepts that can be introduced at the UG level are selected and instead of cramming the course with too many ideas the stress is given in doing the selected concepts rigorously. The idea is to make learning mathematics meaningful and an enjoyable activity rather than acquiring manipulative skills and reducing the whole thing an exercise in using thumb rules.

As learning Mathematics is doing Mathematics, to this end, some activities are prescribed to increase student's participation in learning. Duration of the degree programme shall be six semesters distributed in a period of three academic years.

**2. Title of the Course:** **B.Sc (Mathematics)**

**3. Objectives of the Course:**

Successful Mathematics students of this institute will gain lifelong skills, including following:

- To develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment.
- The development of their mathematical knowledge.
- Confidence by developing a feel for numbers, patterns and relationships.
- An ability to consider and solve problems and present and interpret results.
- Communication and reason using mathematical concepts.
- To develop an understanding of mathematical principles.
- To develop the abilities to reason logically, to classify, to generalize and to prove.
- To acquire a foundation appropriate to their further study of mathematics and of other disciplines.

**4. Advantages of the Course:**

Student will be getting highly motivated for higher studies in reputed institutes like IIT's , NIIT's,IISc's,CMI,HRI etc...

| | |
|---|---|
| **5. Duration of the Course:** | Three years |
| **6. Eligibility of the Course:** | **B.Sc. II(One optional as Mathematics )** |
| **7. Strength of the Students:** | 40 |
| **8. Fees for Course:** | As per UGC/University/College rules. |
| **9. Period of the Course:** | As per UGC/University/College rules |
| **10. Admission / Selection procedure:** | As per UGC/University/College rules |
| **11. Teacher's qualifications:** | As per UGC/University/College rules |
| **12. Standard of Passing:** | As per UGC/University/College rules |
| **13. Nature of question paper with scheme of marking:** | As per UGC/University/College rules |
| 15**. List of book recommended**: | Included in syllabus |
| 16. **List of Laboratory Equipments, Instruments, Measurements etc.:** | ------ |
| 17. **Rules and regulations and ordinance if any:** | As per UGC/University/College rules |
| **18. Medium of the language:** | English |
| **19. Structure of the Course:** | Attached as Annexure 'A' |
| **20. Allotment of workload** (Theory/Practical): | Attached as Annexure 'A' |
| **21. Staffing pattern:** | As per UGC/University/College rules. |
| **22. Intake capacity of students:** | As per UGC/University/College rules |
| **23. Paper duration:** | Each theory paper is of 45Contact hours |
| **24. To be introduced from:** | **B. Sc. III from June 2020** |

Chairman Board of Studies
Mathematics

**(Mr. M. S. Wavare)**

**List of BoS Member(2019-2022)**

1. Dr. D D Pawar (VC Nominee)
   Director,
   School of Mathematical Sciences
   Swami Ramanand Teerth Marathwada University,
   Nanded.
2. Dr. S D Kendre (Subject Expert)
   Department of Mathematics,
   SPPU,Pune
3. Dr. M T Gophane (Subject Expert)
   Department of Mathematics
   Shivaji University ,Kolhapur .
4. Dr. A A Yadav
   R S M , Latur
5. Prof .S M Shinde (Student Alumni)
   Govt.College of Engg., Karad Dist :Satara
6. Mr. S S Ranmal
   Sungrace Computers Pvt Ltd, Pune
7. Prof. N . S. Pimple
   R S M , Latur
8. Dr. S B Birajdar
   R S M , Latur
9. Mr. S D Malegaokar
   R S M , Latur
10. Miss A B Kale
    R S M , Latur
11. Mr. D M Ghuge
    R S M , Latur

# Program Outcomes :

On Successful UG Mathematics students of this institute will gain lifelong skills, including following:

- To develop their mathematical knowledge and oral, written and practical skills in a way which encourages confidence and provides satisfaction and enjoyment.
- Confidence by developing a feel for numbers, patterns and relationships.
- An ability to consider and solve problems and present and interpret results.
- Communication and reason using mathematical concepts.
- To understanding of mathematical principles and their applications
- To develop the abilities to reason logically, to classify, to generalize and to prove.
- To acquire a foundation appropriate to their further study of mathematics and of other disciplines.
- They Can qualify IIT-JAM entrance in the subject of Mathematics
- They will get admitted to ranked institute in India
- They can do minor research in the field of Maths
- They can solve problems independently

**Annexure 'A'**

# Rajarshi Shahu Mahavidyalaya, Latur (Autonomous)

## Department of Mathematics

**B.Sc. III (Mathematics) Semester V**

**Curriculum Structure with effect from June, 2020**

| Code No. | Title of the course with paper number | Hours/ Week | Marks (50) | | Credits |
|---|---|---|---|---|---|
| | | | In Sem | End Sem | |
| DSEE-I U-MAT-555 | Metric Space | 03 | 20 | 30 | 02 |
| DSEE-IIU-MAT-556(A) OR DSEE-IIU-MAT-556(B) | Linear Algebra Or Multivariable Calculus | 03 | 20 | 30 | 02 |
| DSEEP-I U-MAT-557 | Laboratory Course On Metric Space | 03 | 20 | 30 | 02 |
| DSEEP-II U-MAT-558(A) Or U-MAT-558(B) | Laboratory Course on Linear Algebra Or Multivariable Calculus | 03 | 20 | 30 | 02 |
| Skill Course SEC-III U-ADC-540-L | Latex Type setting | 03 | 20 | 30 | 02 |
| | Total Credits | | | | 10 |

**Student Stay Hours: 15/Week**

**B.Sc.III (Mathematics) Semester VI**

| Code No. | Title of the course with paper number | Hours/ Week | Marks (50) | | Credits |
|---|---|---|---|---|---|
| | | | In Sem | End Sem | |
| DSEE-III U-MAT-645 | Complex Analysis | 03 | 20 | 30 | 02 |
| DSEE-IV U-MAT-646 (A)<br>Or<br>DSEE-IV U-MAT-646(B) | Theory of Probability and Distributions<br>Or<br>Number Theory | 03 | 20 | 30 | 02 |
| DSEEP-III U-MAT-647 | Laboratory Course On Complex Analysis | 03 | 20 | 30 | 02 |
| DSEEP-IV U-MAT-648(A)<br>Or<br>DSEE-IV U-MAT-648(B) | Laboratory Course On Theory of Probability and Distributions<br>Or<br>Number Theory | 03 | 20 | 30 | 02 |
| Skill Course SEC-IV | Python Programming | 03 | 20 | 30 | 02 |
| | Total Credits | | | | 10 |

**Student Stay Hours: 15/Week**

**B. Sc. –III [Mathematics] Semester V**

**Course Code: U-MAT-555**

## Paper-IX Metric Space

**Learning Objectives:**

- Metric, Neighborhood, Limit Point, Open Set, Closed Set.
- Cauchy Sequence, Complete Metric spaces Compactness & Connectedness.
- Weierstrass theorem, Lebesgue Covering lemma ,Continuity and Uniform Continuity.

**Course Outcomes:**

**On completion of this course successfully students will be able to:**

- Deal with various examples of metric spaces;
- have some familiarity with continuous maps;
- work with compact sets in Euclidean space;
- work with completeness and connectedness;
- apply the ideas of metric spaces to other areas of mathematics.

**Unit I**

Metric Space, Introduction, Metric, Neighborhood, Limit Point, Isolated Point Closed Set, Boundary Sets, Interior point, Interior, Open Set. [10 Lectures]

**Unit-II**

Cauchy Sequence, Complete Metric spaces, Baire category Theorem, Compactness & Connectedness. [18 Lectures]

**Unit-III**

Weierstrass Theorem, Sequentially Compactness, Totally boundedness, Lebesgue number, Lebesgue Covering lemma, Continuity and Uniform Continuity. [17 Lectures]

**Reference Books**

1. Lectures on Analysis by T M Karade, J N Salunke, K S Adhav, M S Bedre ,Sonu-NiluPub.
2. Advanced Calculus (Third Edition) By Robert Wrede & Murray R. Spiegel
3. Mathematical Analysis ,By S.C.Malik & Savita Arora
4. Methods of Real Analysis ,By R. R. Goldberg
5. Elements of Real Analysis, By Shanti Narayan M. D. Raisinghaniya ( S.Chand & Comp. Ltd.)

**Course Code: U-MAT-556(A)**

## Paper –X(A) Linear Algebra

**Learning objectives :**

- Vector space
- Finding dimensions of vector space
- Inner Product space
- Dimension Theorem

**Course Outcomes:**

**After successful completion of this course students can**

- Find dimension basis of vector space over real numbers .
- Apply Gram Schmidt process to find orthonormal basis .
- Find standard matrix of any linear transformation.

**Unit-I Vector Space** [15 Lectures ]

Properties of Vector operations in $R^n$ , Euclidean N Space. Norm and distance in n-space**,** Vector Space definition, examples and simple properties. Subspace, solution space of homogeneous systems, Linear Combination of vectors, linear span of Vectors. Linear dependence and independence, Basis and Dimension .Coordinate to basis, Row space, column space and null space (only statements), Rank-nullity for Matrices (only statements)

**Unit II Inner Product Space** [15 Lectures]

Definition and Examples, Length and distance in inner product space, properties. Cauchy-Schwarz inequality, Properties of Length and distances in inner product space, Angle between vectors, orthogonality, Orthogonal and orthonormal bases, co-ordinate relative to orthogonal and orthonormal bases, Gram-Schmidt methods (Examples only)

**Unit III Linear Transformation** [15 Lectures]

Definition and Example of Linear transformations, properties, Kernel and range of linear transformation .Dimension theorem of Linear Transformation .Linear Transformation from $R^n$ to $R^m$ ,Linear Transformation from images of basis vectors ,All Linear transformations are matrix transformation, Standard matrices of linear transformations.

**Reference Books**

1. Introductory Linear Algebra By T M Karade J N Salunke , Sonu Nilu Publication
2. Linear algebra- a geometric approach by Kumaresan, Prentice-hall of India private limited.

3. Linear algebra by Hoffman and Kunze, Tata McGraw-Hill, New Delhi

4. Modern Algebra, By A.R. Vasishtha, Krishna Prakashan Media.

5. Modern Algebra, By R.P. Rohtatgi, Dominant Publishers and Distributors, New Delhi.

6. Linear Algebra Vivek Sahai Vikas Bist (Second Edition) Narosa

7. Abstract and Linear Algebra Singh ,Pundir,Singh ,Pragati Prakashan

8 Introduction to linear algebra by Serge Lang, Springer verlag

9. First Course in Abstract Algebra, By Vijay K.Khanna,and M L Bhambri Vikas Publication Company

10. An Introduction to Linear Algebra by Krishnamurty, Manaro and Arora.

**Course Code: U-MAT-556(B)**

## Paper –X(B) Multivariable Calculus

**Learning Objectives:**

- **Simultaneous limits and repeated limits evaluation**
- **Continuity of multivariate functions**
- **Series expansion of function**
- **Extreme values of the function**
- **Multiple Integral**

**Course Outcomes:**

**After successful completion of the course students are able to**

- **Evaluate simultaneous limit of multivariate functions**
- **Evaluate continuity of multivariate functions.**
- **Find series representation of given function.**
- **Calculate Maxima and minima of function.**
- **Finding Volume using multiple inegral**

**Unit-I**

Functions of two and three variables, Notions of limits and continuity , Examples Partial Derivatives Definition and examples Chain Rules Differentiability sufficient conditions for differentiability.Higher ordered partial derivatives. [15 Lectures ]

**Unit-II**

Schwartz's theorem, Young's theorem with proof. Euler's theorem for homogeneous functions. Mean Value theorem, Taylor's theorem for functions of two variables Extreme Values Extreme values of functions of two variables.Necessary conditions for extreme values. Sufficient conditions for extreme values. Lagrange's method of undetermined coefficients.[15 Lectures]

**Unit-III**

Multiple Integrals Double integrals, evaluation of double integrals.Change of order of integration for two variables.Double integration in Polar co-ordinates.Triple integrals.Evaluation of triple integrals.Jacobians, Change of variables.(Results without proofs)Applications to Area and Volumes.[15 Lectures ]

**Reference Books**

1. Shanti Narayan and P.K. Mittal, A Course of Mathematical Analysis
(12th Edition, 1979), S. Chand and Co..
2. M.R. Spiegel, Advanced Calculus: Schaum Series.
3. D.V. Widder, Advanced Calculus (IInd Edition), Prentice Hall of India, New Delhi, (1944).
4. T.M. Apostol, Calculus Vol. II (IInd Edition), John Willey, New York, (1967).

**Course Code U-MAT-557**
**Laboratory Course-VII**

## Problems in Metric Space

**Learning Objectives:**

- Metric, Neighborhood, Limit Point, Open Set, Closed Set.
- Cauchy Sequence, Complete Metric spaces Compactness & Connectedness.
- Weierstrass theorem, Lebesgue Covering lemma ,Continuity and Uniform Continuity.

**Course Outcomes:**

**On completion of this course successfully students will be able to:**

- Deal with various examples of metric spaces;
- have some familiarity with continuous maps;
- work with compact sets in Euclidean space;
- work with completeness and connectedness;
- apply the ideas of metric spaces to other areas of mathematics.

# List of Practical's

1. Let (X,d) be a metric space, a, b, c are points in X and A be a subset of X them prove that
   a. $|d(x,z) - d(y,z)| \le d(x,y)$
   b. $|d(x,A) - d(y,A)| \le d(x,y)$
2. Let X be any set. Then the mapping d of XxX into R is a metric iff the following conditions are satisfied
   a. $d(x,y) = 0 \ iff\ x = y \ \forall x, y \in X$
   b. $d(x,y) \le d(x,z) + d(y,z) \ \forall x, y, z \in X$
3. For $1 < p < \infty$ we denote by $l^p$ the space of all sequences $<x_n>$ such that $\sum_{n=1}^{\infty} |x_n|^p < \infty$ let $d: l^p \times l^p \rightarrow \boldsymbol{R}$ be defined by setting $d(x,y) = (\sum_{n=1}^{\infty} |x_i - y_i|)^{\frac{1}{p}}$ where x $=<x_n> \in l^p$ and y $=<y_n> \in l^p$ . Then show that $(l^p,d)$ is a metric space.
4. Describe open spheres of unit radius about (0,0) for each of the following metrics for $R^2$
   a. $d_1(z_1, z_2) = \{(x_1 - x_2)^2 + (y_1 - y_2)^2\}^{1/2}$
   b. $d_2(z_1, z_2) = |x_1 - x_2| + |y_1 - y_2|$
   c. $d_3(z_1, z_2) = \max(|x_1 - x_2| + |y_1 - y_2|)$ Where
   $z_1 = (x_1, y_1), z_2 = (x_2, y_2)$ are any two points of $R^2$
5. Prove that every open sphere is open set as well as every closed sphere is closed set.
6. Show that in a metric space every finite set is closed.
7. Prove that every subset of a discrete metric space is clopen. Is the discrete metric space connected?

8. Show that a subset of a metric space is open iff it is the union of a family of open spheres.
9. State and prove Housdroff theorem.
10. Show that a subset of a metric space is open iff it is neighbourhood of each of its points.
11. Prove that a point p of a metric space is limit point of a subset A iff every neighbourhood of p contains infinitely many points of A.
12. Give an example to show that the inclusion in the result $\overline{S \cap T} \subseteq \bar{S} \cap \bar{T}$ can be strict.
13. Let X be a metric space and Y be its subspace. Then prove that every subset closed in Y is closed in X iff Y is closed in X and every subset open in Y is open in X iff Y is open in X
14. Let $(X,d_1)$ and $(Y,d_2)$ be two metric spaces. Define $d: X \times Y \to \boldsymbol{R}$ such that $d(u,v)=d_1(x_1,x_2)+d_2(y_1,y_2)$ whrer $u=(x_1,y_1)$ and $v=(x_2,y_2)$. Show that d is a metric.
15. Let (X,d) be a metric space, $x_0$ be a point in X and A be a subset of X them prove that the real valued functions $f(x)=d(x_0,x)$ and $g(x)= d(A,x)$ are continuous.
16. State and prove Urysons lemma for metric spaces
17. Let X and Y be two metric spaces and $f: X \to Y$ be one one onto. Then show that the following statements are equivalent
    a. $f^{-1}$ is continuous
    b. For each open set A in X, the set f(A) is open in Y
    c. For each closed set B in X, the set f(B) is closed in Y
18. Show that $f: \boldsymbol{R} \to ]-1,1[$ defined by $f(x) = \frac{1}{1+|x|}$ is a homeomorphism.
19. Show that every function defined on discrete metric space is continuous.
20. Let X denote the metric space **R** with discrete metric and Y denote the metric space **R** with usual metric. Define $f: X \to Y$ and $g: Y \to X$ such that f(x)=x, g(x)=x. Show that f is continuous but g is not.
21. Prove that every convergent sequence in a metric space is Cauchy sequence. Is the converse true? If not can you give a sufficient condition for a Cauchy sequence to become convergent.
22. Let Y be a subspace of a metric space X. Then show that Y is complete iff Y is closed.
23. Show that the following metric spaces are complete
    a. Z with usual metric
    b. [0,1] with the metric $d(x,y) = \frac{1}{1-x} - \frac{1}{1-y}$
24. Show that the the Euclidean space $R^n$ is complete.
25. Prove that every compact metric space is totally bounded ( using only the definition of compactness and totally boundedness of a metric space.) . Is the converse true?
26. List all compact subsets of an discrete metric
27. Let $\{A_\alpha : \alpha \epsilon \Delta\}$ be a family of connected subsets of a metric space such that $\cap \{A_\alpha : \alpha \epsilon \Delta\} \neq \emptyset$. Prove that $\cup \{A_\alpha : \alpha \epsilon \Delta\}$ is connected.
28. Give an example of continuous bijection which is not homomorphism.
29. If there exist a continuous bijection from a metric space X to a discrete space then prove that X is discrete space ( every subset of X is open).
30. Let $f: X \to Y$ be a bijective continuous function. If X is compact then prove that f is homeomorphism. Give an example of closed bounded set which is not compact

**Course Code: U-MAT-558(A)**
**Laboratory Course-VIII (A)**

# Problems in Linear Algebra

**Learning Objectives:**

- Vector spaces and subspaces examples.
- Examples of linear transformation and matrix representation of linear transformation.
- Examples on Inner product spaces.

**Course Outcomes:**

**On completion of this course successfully students will be able to:**

- Deal with various example on vector spaces, Subspaces and Basis.
- Handle the problems on Linear Transformation and matrix representation of linear transformation.
- Discuss the inner product spaces examples and its properties.
- apply the ideas of linear spaces to other areas of mathematics.

**List of Practical's**

1)In an vector space V, Show that (a+b)(x+y)=ax+ay+bx+by.s

2)Prove that tr(xA+yB)=x tr(A)+y tr(B) for any $A,B \in M_{nxn}(F)$.

3)Show that the set of nxn matrices having trace equal to zero is a subspace of $M_{nxn}(F)$.

4)Any intersection of subspaces of a vector space V is a subspace of V.

5)Let $W=\{(x,y,z) \in R^3: x=3y \text{ and } z=-y\}$ then show that W is a subspace of $R^3$.

6)Show that the set $\{1,x,x^2,x^3,\ldots.,x^n\}$ is linearly independent in $P_n(F)$.

7)Determine whether the vectors u=(1,2,3), v=(0,0,1) and w=(0,1,2) span in $R^3$.

8)Let u=(1,1,1), v=(1,1,0) and w=(1,0,0) be the three vectors in $R^3$. Show that (3,2,1) is linear combination of u,v and w.

9)Let u,v and w be distinct vectors of a vector space V.Show that if {u,v,w} is a basis for V, then {u+v+w,v+w,w} is also a basis for V.

10)Let the set $\{1-2x-2x^2,-2+3x-x^2,1-x+6x^2\}$ is basis for $P_2(R)$.

11) Define $T:R^2 \rightarrow R^2$ by $T(a_1,a_2)=(2a_1+a_2,a_1)$ then show that T is linear.

12)Define $T:M_{mxn}(F) \rightarrow M_{nxm}(F)$ by $T(A)=A^t$ ,where $A^t$ is the transpose of A.
Show that T is a linear transformation.

13) Let $T:R^2 \rightarrow R^2$ be the linear transformation defined by $T(a_1,a_2)=(a_1+a_2,a_1)$. Then find N(T).

14)Define $T:P_2(R)\rightarrow P_3(R)$ by $T(f(x))=2f'(x)+\int_0^x 3f(t)\,dt$ then show that T is linear transformation.

15)Let V and W be vector spaces, and let $T:V\rightarrow W$ be linear. Then prove that T is one–to-one if and only if $N(T)=\{0\}$.

16)Define $T:F^2\rightarrow P_1(F)$ by $T(a_1,a_2)=a_1+a_2x$. Then show that T is an isomorphism.

17) Let $T:R^2\rightarrow R^3$ be the linear transformation defined by $T(a_1,a_2)=(a_1+3a_2,0,2a_1-4a_2)$. Then find matrix representation of T with respect to standard order bases for $R^2$ and $R^3$.

18) Let $T:P_3(R)\rightarrow P_2(R)$ be the linear transformation defined by $T(f(x))=f'(x)$. Then find matrix representation of T with respect to standard order bases for $R^2$ and $R^3$.

19) Let V and W be vector spaces over a field F, and let $T,U:V\rightarrow W$ be linear. Then prove that aT+U is linear.

20)Let $V=P_2(R)$ and let $B=\{1,x,x^2\}$ be the standard ordered basis for V.

If $f(x)=4+6x-7x^2$ then find $[f]_B$.

21)Let B be a basis for a finite-dimensional inner product space. Prove that if $<x,z>=<y,z>$ for all $z\epsilon B$ then x=y

22) Show that set {(1,1,0),(1,-1,1),(-1,1,2)} is an orthogonal set in $F^3$.

23) Let x=(2,1+i,i) and y=(2-i,2,1+2i) be vectors in $C^3$. Compute $<x , y>$,$\|x\|$ ,$\|y\|$,$\|x+y\|$.

24) Obtain an orthonormal basis w.r.t. the standard inner product for the subspace of $R^3$ generated by (1,0,3) and (2,1,1).

25) Let $u=(x_1, x_2)$ and $v=(y_1,y_2)\in R^2$.

(i) Verify that the following is an inner product space on $R^2$

$(u,v)=x_1y_1-2x_1y_2-2x_2y_1+5x_2y_2$.

(ii) For what values of k is the following an inner product space on $R^2$

26) Consider standard inner product on $R^2$ .

Suppose $\alpha=(1,2)$, $\beta=(-1,1)\in R^2$ . If $\gamma\in R^2$ be s.t. $(\alpha,\gamma)=-1$ and $(\beta,\gamma)=3$ . find $\gamma$.

27) Show that for any $\alpha\in R^2$

$\alpha=(\alpha,\epsilon_1)\epsilon_1+(\alpha,\epsilon_2)\epsilon_2$ , where $\epsilon_1=(1,0)$ , $\epsilon_2=(0,1)$ .

28) Let $\alpha_1$=(1,1,-2,1) , $\alpha_2$=(3,0,4,-1) , $\alpha_3$=(-1,2,5,2) . Show that the vector (4,-5,9,-7) is spanned by $\alpha_1$ , $\alpha_2$ , , $\alpha_3$ .

29) If W is a subspace of V and $v\in V$ satisfies $(v, w)+(w,v)\leq(w, w)$ for all $w\epsilon W$.

30)If Vis a finite dimensional inner product space and $f\epsilon V$. Prove that there exist $u_0\epsilon V$ s.t. $f(v)=<v,u_0>$ for all $v\epsilon V$. Also show that $u_0$ is uniquely determined.

31) Obtain an orthonormal basis, w.r.t. the standard inner product for the subspace of $R^3$ generated by (1,0,3) and (2,1,1) using Gram-Schmidt Process.

32) Using Cauchy Schwarz inequality, prove that cosine of an angle is of absolute value at most 1.

**Course Code: U-MAT-558(B)**
**Laboratory Course-VIII (B)**

# Problems in Multivariable Calculus

**Learning Objectives:**

- **Simultaneous limits and repeated limits evaluation**
- **Continuity of multivariate functions**
- **Series expansion of function**
- **Extreme values of the function**
- **Multiple Integral**

**Course Outcomes:**

**After successful completion of the course students are able to**

- **Evaluate simultaneous limit of multivariate functions**
- **Evaluate continuity of multivariate functions.**
- **Find series representation of given function.**
- **Calculate Maxima and minima of function.**
- **Finding Volume using multiple inegral**

1. Give the domain of definition for which each of following functions is defined and real and indicate this domain graphically
   a) $f(x,y) = \ln\{(16 - x^2 - y^2)(x^2 + y^2 - 4)\}$
   b) $f(x,y) = \sqrt{6 - 2x + 3y}$
   c) $f(x,y) = \frac{1}{x^2+y^2-1}$
   d) $ln(x + y$
2. Using definition of limit ,prove that
   a) $\lim_{(x,y)\to(1,2)} x^2 + 2y = 5$
   b) $\lim_{(x,y)\to(4,1)} 3x - 2y = 14$
   c) $\lim_{(x,y)\to(2,1)} xy - 3x + 4 = 0$
3. Using definition prove that f(x,y)=xy+6x is continuous at (1,2) .

4. Show that limit when $(x,y) \to (0,0)$ does not exist for $f(x,y) = \frac{2xy}{x^2+y^2}$.

5. Show that limit when $(x,y) \to (0,0)$ does not exist for $f(x,y) = \frac{xy^3}{x^2+y^6}$.

6. Show that $\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2} = 0$.

7. Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$.

8. If $f(x,y) = \frac{xy}{x^2+y^2}$ ,then show that repeated limit exists but are not equal when $((x,y) \to (0,0)$.

9. If $f(x,y) = \frac{y-x}{y+x}\frac{1+x}{1+y}$ then show that repeated limit exists but not equal when $(x,y) \to (0,0)$.

10. Show that the repeated limit exist but the double limit does not when $(x,y) \to (0,0)$

    a) $f(x,y) = \frac{x-y}{x+y}$

    b) $f(x,y) = \frac{x^2y^2}{x^4+y^4-x^2y^2}$

11. If $f(x,y) = 2x^2 - xy + y^2$ then find i) $\frac{\partial f}{\partial x}$ at $(x_0,y_0)$ directly from definition ii) $\frac{\partial f}{\partial y}$ at $(x_0,y_0)$ directly from definition

12. If $\emptyset(x,y) = x^3y + e^{xy^2}$ then find i) $\emptyset_x$ ii) $\emptyset_y$ iii) $\emptyset_{xx}$ iv) $\emptyset_{yy}$ v) $\emptyset_{xy}$ vi) $\emptyset_{yx}$

13. If $f(x,y) = \sqrt{|xy|}$ then find $f_x(0,0)$ and $f_y(0,0)$

14. Show that $U(x,y,z) = (x^2+y^2+z^2)^{(-1/2)}$ satisfies Laplace's partial differential equation $\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0$. We assume here that $(x,y,z) \neq (0,0,0)$.

15.If $Z=x^2\tan^{-1}(y/x)$, then find $\frac{\partial^2 Z}{\partial x \partial y}$ at (1,1).

16. If $f(x,y)=\begin{cases} \frac{x^3-y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$, then show that function is not differentiable at the origin.

18. Show that funtion f, where $= \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & if\ x^2+y^2 \neq (0,0) \\ 0, & if\ \ x = y = 0 \end{cases}$ is continuous, posseses partial derivative but not differentiable at the origin.

19. Let $U=x^2e^{y/x}$ find dU.

20. Show that $(3x^2y-2y^2)dx+(x^3-4xy+6y^2)dy$ can be written as an exact differential of a function $\varphi(x,y)$ and find this function.

21. Let $=\frac{xy(x^2-y^2)}{x^2+y^2}$, (x,y)$\neq$ (0,0), f(0,0)=0 , then show that at the origin $f_{xy} \neq f_{yx}$.

22. Find the maxima and minima of the function $f(x,y)$ =$x^3+y^3-3x-12y+20$.

23. Find critical points of the function $f(x,y)$ =$x^3+y^2-12x-6y+40$ test each of these for maxima and minima.

24. A rectangular box open at the top is to have volume of 32 cu.ft. what must be the dimensions so that the total surface is minimum

25.Evaluate

a. $\int_0^1 \int_{0y}^1 x^2 e^{xy}\, dxdy$ *(IIT-JAM 2006)*

b. $\int_0^2 \int_0^{x^2} e^{\frac{y}{x}}\, dxdy$

c. $\int_0^a \int_0^{\sqrt{ay}} xy\, dxdy$

d. $\int_0^a \int_0^{\sqrt{a^2-y^2}} dxdy$

26. Using the transformation $x+y=y, y=uv$ such that $\iint [xy(1-x- \quad y)]^{\frac{1}{2}}\, dxdy = \frac{2\pi}{105}$

27. Let V be the region bounded by planes x=0,x=2,y=0,z=0 and y+z=1 then find the value of integral $\iiint_V y dxdydz$ *(IIT-JAM 2011)*

28. Find the volume of region in the first octant bounded by surfaces $x=0, y= \quad x, y=2-x^2\ z=0\ and\ z=x^2$ *(IIT-JAM-2010)*

29 Evaluate $\iiint_W zdxdydz$ ,where W is the region bounded by the planes $x=0,\ \ y=0,\ z=o\ ,z=1\ and\ the\ cylinder\ x^2+y^2=1\ with\ x\geq 0\ y\geq 0$ *(IIT-JAM 2006*

30. Evaluate the integral $\iiint (x^2+y^2+z^2)dxdydz$ taken over volume enclosed by sphere $x^2+y^2+z^2=1$

## Skills Enhancement Course-IV

## Latex Typesetting U-ADC-540-L

**Learning Objectives :**

- Latex Installation
- Layout Design
- Packages
- Mathematical Symbols and equations

**Course Outcomes**

After completing this course students are able to

- Learn different environment in Tex
- Learn how to input maths symbol and equation

**Unit I:** Introduction to LaTeX, Installation of LaTeX, Layout Design, LaTeX input files, Input file structure, document classes, packages, environments, page styles,

**Unit II:** Inline math formulas and displayed equations, Math symbols and fonts, Delimeters, matrices, arrays, Typesetting Mathematical formulae: fractions, Integrals, sums, products,

**Reference Books :**


1. Latex Tutorials Indian Tex user group Trivendrum India
2. Latex line by line Tips and Techniques for document processing - Antoni Diller


**List of Practical's**

**1.Writing Research Article using Tex .**

**2.Writing Application letter Using Tex**

**3.Creating Matrix Using Tex**

**4.Creating Table using Tex**

**5.Creating PPT using Tex**

**6. Mathematics Formulae in Tex**

**B. Sc. –III [Mathematics] Semester VI**

**Course Code: U-MAT-655**

# Paper-XI Complex Analysis

**Learning Objectives:**

- Cauchy-Goursat's Theorem, Cauchy integral formula
- Liuouville's Theorem. Fundamental Theorem of Algebra
- Taylor Series, Laurent Series, Absolute and uniform convergence of power series.
- Residues, Cauchy residue theorem, Applications of Residues.

**Course Outcomes:**

**On completion of this course successful students will be able to:**

- Understand the significance of differentiability for complex functions and be familiar with the Cauchy-Riemann equations.
- Evaluate integrals along a path in the complex plane and understand the statement of Cauchy's Theorem.
- Compute the Taylor and Laurent expansions of simple functions, determining the nature of the singularities and calculating residues.
- Use the Cauchy Residue Theorem to evaluate integrals and sum series.

**Unit -I Functions of Complex Variables** [15 Lectures]

Definitions and examples, Limit , Theorems on limit, Continuity, Derivative , Differentiable functions, Algebra of differentiable functions. Chain rule, Cauchy Riemann equations, Sufficient conditions, C-R equations in polar form, formula for $f'(z_0)$ .Analytic functions, Harmonic functions and harmonic conjugate. Elementary functions, Bilinear transformation.

**Unit II Integrals** [15 Lectures]

Contour, Simple arc, line integral, Proof of the result $\left| f(z) \cdot dz \right| \leq ML$, Cauchy-Goursat's theorem, Simply and multiply connected domains, Cauchy integral formula. Derivatives of analytic functions, Taylors series and Laurent series, examples, Liuoville's theorem. Fundamental theorem of Algebra.

**Unit III Singularities and the Calculus of Residues** [15Lectures]

Zero of function, singular point, , Different types of singularities , Limiting point of zero and poles, some theorems, Definition of Residue, Cauchy residue theorem, Principal part of a function, poles and residues at poles, Applications of residues Evaluation of improper integrals, examples.

**Reference Books**

1. R.V. Churchill and I.W. Brown, Complex Variables and Applications, International Student Edition, 2003. (Seventh Edition).

2.S. Ponnusamy, Complex Analysis , Second Edition (Narosa).

3. J.M. Howie, Complex Analysis, (Springer, 2003).

4. S. Lang, Complex Analysis, (Springer Verlag).

5. A.R. Shastri, An Introduction to Complex Analysis, (MacMillan).

6. Lectures on Analysis by T M Karade, J N Salunke, K S Adhav, M S Bedre ,Sonu-NiluPub

7. Goyal & Gupta, Functions of a Complex Variable, A Pragati Edition,Meerut.

8. Shanti Narayan & Dr. P. L. Mittal , Theory of Functions of a Complex Variable, S. Chand & Company Ltd.Ram Nagar,New Delhi.

**Course Code: U-MAT-656(A)**
**Paper-XII(A)**

# Theory of Probability and Distributions

**Learning objectives**

- **Elementary theory of probability**
- **Discrete and continuous random variable**
- **Discrete probability distribution**
- **Continuous probability distribution**

**Course Outcomes:**

**Students are able to**

- **Solve examples on Bays Theorem**
- **Differentiate continuous and discrete random variable**
- **Apply discrete probability distributions**
- **use Normal Distribution**

**Unit-I: Basics of Probability**

Basic Definitions, Mathematical and statistical probability, Axiomatic approach to probability, Theorems on probability, Conditional probability with examples, Extended axiom of addition and continuity, Baye's theorem. [10 lectures]

**Unit-II: Random Variables**

Random variables, Types - discrete random variable, Continuous random variable, probability distribution function, probability density function, Mathematical expectation, Properties of expectation and Variance, Moment generating function, Cumulant generating function, Probability generating function, and its properties. [15 lectures]

**Unit-III: Some Discrete and Continuous Distributions**

Discrete Probability distributions: Binomial distribution, Poisson distribution, Discrete Uniform distribution, Hypergeometric distribution; its Mean and Variance; MGF and CGF of distributions, Fitting of distributions and its applications. Continuous Probability distributions: Normal distribution, Exponential distribution, its properties, Moments and applications. [20 lectures]

**Reference Books**


1. Probability and Statistics for Engineer and Scientist, by Miller and Freund's Johnson RA,PHI
2. Fundamental of Mathematical Statistics,by S.C. Gupta, V.K. Kapur, S. Chand and Co. Ltd.
3. Mathematical Statistics, by S. C. Saxena, S. Chand and Co. Ltd.
4. Modern Theory of probability ,by B R Bhat ,Narosa
5. Theory of Probability by P Mukhopadhyay,Narosa.

**Course Code: U-MAT-656(B)**
**Paper-XII(B)**
**Number Theory**

**Learning Objectives:**

- **Elementary number theory**
- **Prime distribution**
- **Arithmetical functions**

**Course Outcomes : Students are able to**

- **Apply Chinese Remainder Theorem**
- **Solve Quadratic Congruence's**
- **Use $\varphi(n)$**

**Unit I:Theory of Congruence's**

Theory of congruences, Basic properties of congruences, Binary and decimal representation of integers, Linear congruences and Chinese Remainder theorem, Pierre de Fermat theorem, Fermat's little theorem and pseudo primes, Wilson's theorem. [15 lecturtes]

**Unit-II: Number Theoretic Functions**

The sum and number of divisors ,The Mobius Inversion Formula ,The greatest integer function, an application to the calendar , Eulers Phi Function , Eulers Theorem and properties. [15 lectures ]

**Unit III: Prime roots and Indices**

The order of an integer modulo n, Primitive roots for primes, Composite numbers having primitive roots, Euler's criterion, The Legendre symbol and its properties, Quadratic reciprocity, Quadratic congruences with composite moduli, [15 lecturtes]

**Reference Books**:

1. David M. Burton, "*Elementary Number Theory*" Tata McGraw-Hill Pub. VI Edition.

2. Tom M. Apostol, "*Introduction to Analytic number theory*" Narosa Publishing house 1980.

3. A course in arithmetic- J.P. Serre. GTM Vol.7, Springer Verlage 1973

**Course Code: U-MAT-657**

**Laboratory Course-IX**

## Problems on Complex Analysis

**Learning Objectives:**

- Cauchy-Goursat's Theorem, Cauchy integral formula
- Liuouville's Theorem. Fundamental Theorem of Algebra
- Taylor Series, Laurent Series, Absolute and uniform convergence of power series.
- Residues, Cauchy residue theorem, Applications of Residues.

**Course Outcomes:**

**On completion of this course successful students will be able to:**

- Understand the significance of differentiability for complex functions and be familiar with the Cauchy-Riemann equations.
- Evaluate integrals along a path in the complex plane and understand the statement of Cauchy's Theorem.
- Compute the Taylor and Laurent expansions of simple functions, determining the nature of the singularities and calculating residues.
- Use the Cauchy Residue Theorem to evaluate integrals and sum series.

## List of Practical's

1. Prove that $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exist.

2. Is the function $f(z) = \frac{3z^4 - 2z^3 + 8z^2 - 2z + 5}{z - i}$ continuous at z= i ?

3. Find all points of discontinuity for the following functions.

a) $f(z) = \frac{2z-3}{z^2+2z+2}$  b) $f(z) = \frac{3z^2+4}{z^4-16}$

4. Show that

i. The complex variable function f(z) = $|z|^2$ is differentiable only at the origin.

ii. f(z) $= z^3$ is analytic in the entire z plane.

iii. f(z) $= z|z|$ is not analytic anywhere.

5. Show that the function $x^2 - y^2 + 2y$ is harmonic remains harmonic under the transformation $z = w^3$.

6. If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find f(z) in terms of z.

7. Determine the analytic function, whose real part is

a) $zx(1-y)$ b) $\log\sqrt{x^2 + y^2}$

c) $3x^2y + 2x^2 - y^3 - 2y^2$ d) $\cos x.\cosh y$

8. Prove that a) $\sin(z_1 + z_2) = \sin z, \cos z_2 + \cos z, \sin z_2$

b) $\cos(z_1 + z_2) = \cos z; \cos z_2 - \sin z, \sin z_2$

9 Find the image of $|z - 2i| = 2$ under the mapping $w = \dfrac{1}{z}$

10. Show that the function $w = \dfrac{4}{z}$ transform the straight line $x = c$ in the z-plane into a circle in the w-plane.

11.Determine the region of the w-plane into which the region bounded by $x = 1, y = 1, x + y = 1$ is mapped by the transformation $w = z^2$.

12. Find the image and draw a rough sketch of the mapping of the region $1 \le x \le 2$ and $2 \le y \le 3$ under the mapping $w = e^z$.

13.Find the bilinear transformation that maps the points $z_1 = -i, z_2 = 0, z_3 = i$ into the points $w_1 = 1, w_2 = i, w_3 = 1$ respectively.

14. Show that the transformation $w = \dfrac{3-z}{z-2}$ transforms the circle with centre $\left(\dfrac{5}{2}, 0\right)$, and radius $\dfrac{1}{2}$ in the z-plane into the imaginary axis in the w-plane and the interior of the circle into the right half of the plane.

15. Evaluate : $\int_0^{1+i} z^2 dz$.

16.Find the value of the integral $\int_0^{1+i} (x - y + ix^2) dz$

i) Along the straight line from z=0 to z=1+I;

ii) Along the real axis from z=0 to z=1 and then along a line parallel to the imaginary axis from z=1 to z=1+i.

17.Using Cauchy integral formula calculate the following integrals :

i) $\int_C \frac{zdz}{(9-z^2)(z+i)}$, *where C is the circle* $|z| = 2$ *described in positive sense.*

ii) $\int_C \frac{dz}{z(z+\pi i)}$, *where C is the circle* $|z+3i| = 1$ *described in positive sense.*

iii) $\int_C \frac{\cosh(\pi z)dz}{z(z^2+1)}$, *where C is the circle* $|z| = 2$.

18. For the function $f(z) = \frac{2z^3+1}{z^2+z}$,find

i) A Taylor's series valid in the neighborhood of the point i.

ii) A Laurent's series valid within the annulus of which centre is origin.

19.Expand: $\frac{1}{z(z^2-3z+2)}$ $for\ (i) 0<|z|<1$ , $(ii)\ 1<|z|<2,\ (iii)\ |z|>2.$

20. Find the Taylor's and Laurent's series which represent the function

$\frac{z^2-1}{(z+2)(z+3)}$, $when\ (i)|z|<2$ , $(ii)\ when\ 2<|z|<3,\ (iii)\ when\ |z|>3.$

21. Prove that the function $e^z$ has an isolated essential singularity at $z=\infty$.

22. Find the singularities of the function $\frac{e^{c/(z-a)}}{e^{z/a}-1}$ indicating the character of each singularity.

23. What kind of singularity have the following functions:

(i) $\sin\frac{1}{1-z}\ at\ z=1$ (ii) $\tan\left(\frac{1}{z}\right)\ at\ z=0$ (iii) $\frac{1}{\cos(1/z)}\ at\ z=0$ (iv) $\operatorname{cosec}\frac{1}{z}\ at\ z=0$.

24. Show that the function $e^{-1/z^2}$ has no singularities.

25. Prove that $\int_0^{2\pi} \frac{d\theta}{a+b\cos\theta} = \frac{2\pi}{\sqrt{a^2-b^2}}, a>b>0.$

26. Show by contour integration that $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}.$

27. Apply the calculus of residues to prove that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3} = \frac{3\pi}{8}.$

28. Evaluate: $\int_0^\infty \frac{dx}{(x^4+a^4)}\ (a>0)$.

29. Apply the calculus of residues to prove that $\int_0^\infty \frac{\sin \pi x dx}{x(1-x^2)} = \pi.$

30.By integrating round a suitable contour, prove that : $\int_0^\infty \frac{(\log x)^2 dx}{(1+x^2)} = \frac{\pi^2}{8}$.

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**Course Code: U-MAT-658(A)**

**Laboratory Course-X(A)**

## Problems on Theory of Probability and Distributions

**Learning objectives**

- **Elementary theory of probability**
- **Discrete and continuous random variable**
- **Discrete probability distribution**
- **Continuous probability distribution**

**Course Outcomes:**

**Students are able to**

- **Solve examples on Bays Theorem**
- **Apply discrete/Continuous probability distributions**
- **Use of Binomial ,Poisons and Normal Distribution**

## List of practical's

1. The probability that a person will get an electric contact is 2/5 & the probability that he will not get plumbing contract is 4/7 . If the probability of getting atleast one contract is 2/3 , what is the probability that he will get both ? .

2.A black and red dice are rolled in order. Find the conditional probability of obtaining
a) A sum greater than 9, given that the black die resulted in a number less than 5.
b) A sum 8, given that the red die resulted in a number less than 4 .

3. Three cards are drawn successively without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and third card drawn is an ace ?.

4. If $P(A) = 3/8$ , $P(B) = 1/2$ & $P(A \cap B) = 1/4$ . Find $P(A | B)$ & $P(B | A)$ .

5. An instructor has test bank of 300 easy true/false questions . 200 difficult true/false questions . 200 difficult true/false questions , 500 easy multiple choice questions (MCQ) & 400 difficult multiple choice questions. If a question is selected at random from the test bank .What is the probability that it will be an easy question given that it is multiple choice question ?

6. A fair coin and an unbiased die are tossed. Let A be the event Head appears on the coin & B be the event 3 on the die. Check whether A & B are independent or not .

7. A can solve 90% of the problems given in a book & B can solve 70%. What is the probability that at least one of them will solve the problem,selected at random from the book ?

8. There are two bags,one of which contain 3 black & 4 white balls while the other contains 4 black & 3 white balls. A fair die is cast ,if the face 1 or 3 turns up, a ball is taken from the first bag , & if any other face returns up a ball is chosen from the second bag. Find the probability of choosing a black ball ?

9. A four digit number is formed using the digits 1,2,3,5 with no repetitions. Write the probability that the number is divisible by 5.

10. If $P(x) = x/15$ , $x=1,2,3,4,5$

$0$ , elsewhere .

Find a) $P(X=1 \text{ or } 2)$

b) $P\{1/2 < X < 5/2 \mid X>1\}$

11. Two dice are rolled . Let X denote the random variable which counts the total number of points on the upturned faces. Construct the table given the nonzero values of the probability mass function & draw the probability chart. Also find the distribution function of X.

12. Let X be a continuous random variable with p.d.f. :

$$f(x)= \begin{cases} ax & , 0 \le x \le 1 \\ a & , 1 \le x \le 2 \\ -ax+3a & , 2\le x\le 3 \\ 0 & , \text{elsewhere} \end{cases}$$

a) Determine constant a .

b) Compute $P(x \le 1.5)$

13. Calculate the standard deviation & mean deviation from mean if the frequency function f(x) has the form :

$$f(x) = \begin{cases} (3+2x)/18 & , 2 \le x \le 4 \\ 0 & , \text{otherwise} \end{cases}$$

14. Verify that the following is a distribution function :

$$f(x ,y)= \begin{cases} 0 & , x < -a \\ (1/2)(x/a + 1) & , -a \le x \le a \\ 1 & , x > a \end{cases}$$

15. A privately owned liquor store operates both a drive-up facility as well as a walk-in facility. On a random selected day, let X & Y respectively , be the proportions of the time that the drive-up & walk-in facilities are in use & suppose that the joint density function of these random variable is given by :

$$f(x,y) = \begin{cases} 2(x+2y)/3, & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find the marginal density of X.
b) Find the marginal density of Y.
c) Find the probability that the drive-up facility is busy less than one half of the time.

16. Let X & Y denote the lengths of life in years, of the components in an electronic system. If the joint density function of these random variable is given by :

$$f(x,y) = \begin{cases} e^{(X+Y)}, & x, y > 0 \\ 0, & \text{Otherwise.} \end{cases}$$

a) Find the marginal density of X.
b) Find the marginal density of Y.

17. The joint probability density function of random variable X,Y & Z is given by :

$$f(x,y,z) = \begin{cases} 4xyz^2/9, & 0 < x, y < 1,\ 0 < z < 3 \\ 0, & \text{elsewhere} \end{cases}$$

a) Find the marginal density function of Y & Z
b) Find the marginal density of Y.
c) $P(1/4 < X < 1/2,\ Y > 1/3,\ 1 < Z < 2)$
d) $P(0 < X < 1/2 \mid Y=1/4, Z=2)$.

18. Determine whether the two random variables of the following joint probability distribution are dependent or independent .

| f(x,y) /x | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 0 | 1/6 | 1/12 |
| 2 | 1/5 | 1/9 | 0 |
| 3 | 2/15 | 1/4 | 1/18 |

19. By investing in a particular stock, a person can make a profit in one year of $4000 with the probability 0.3 or take a loss $1000 with the probability 0.7. What is this person expected gain ?

20. A continuous random variable X has the density function :

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

21. A traffic control engineer reports that 75% of the vehicle passing through a check point, are from within the state . What is the probability that fewer that 4 of the next vehicle are from out of the state ? Also find mean & variance

22. The probability that a student pilot passes the written test for his private pilots license is 0.7. Find the probability that a person passes the test

a) on the third try

b) before the fourth try.

23. Suppose that the number of births resulting in twins during year has a Poisson distribution with (parameter) mean one. Calculate the probability that during a year there is

i) no twin birth

ii) exactly one birth

iii) less than two twin birth

iv) greater than one twin birth .

24. A soft drink machine is regulated so that it discharges an an averages of 200ml per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 ml :

a) What fraction of ther cups wil contain more than 224 ml ?

b) What is the probability that a cup contains between 191 & 209 ml ?

c) How many cups will prababily overflow if 230 ml cups are used for the next 1000 drinks ?

25. Given a standard normal distribution, find the value of k such that

a) $P(Z< k)=0.0427$

b) $P(Z> k)=0.2946$

c) $P(-0.93< Z < k)=0.7235$

26. Use the gamma function with $y = 2x$ to show that $\Gamma(1/2) = \pi$ .

27. If a random variable X has the gamma distribution with $\alpha = 2$ & $\beta = 1$ , Find $P (1.8 < x < 2.4)$ .

28. In a certain city, the daily consumption of electric power, in millions of kilowatt-hours is a random variable X having a gamma distribution with mean $\mu = 6$ & variance $\sigma^2 =12$ :

a) Find the value of $\alpha$ & $\beta$ .

b) Find the probability that on any given day the daily consumption of electric power will exceed 12 million kilowatt-hours.

29. A random variable X has a mean $\mu = 10$ & a variance $\sigma^2 =4$. Using Chebyshev' s theorem . Find

a) $P (|X - 10| \geq 3)$

b) $P (|X - 10| < 3)$

c) $P (5 < X < 15)$

d) the value of the constant c such that $P (|X - 10| \geq c) \leq 0.04$ .

**Course Code: U-MAT-658(B)**

**Laboratory Course-X(B)**

# Problems on Number Theory

**Learning Objectives :**

- **Elementary number theory**
- **Prime distribution**
- **Arithmetical functions**

**Course Outcomes : Students are able to**

- **Apply Chinese Remainder Theorem**
- **Solve Quadratic Congruence's**
- **Use $\varphi(n)$**

## List of practical's

**1.** Show that 41 divides $2^{20} - 1$.

**2.** Find the remainder obtained upon dividing the sum 1! + 2! + 3! + 4! + … + 99! + 100! By 12.

**3** Give an example to show that $a^2 = b^2$ (mod n) need not imply that a= b (modn).

4. If a= b (mod n), prove that gcd(a, n) = gcd(b, n).

5.Find the remainders when $2^{50}$ and $41^{65}$ are divided by 7.

6.What is the remainder when the following sum is divided by 4?
$1^5 + 2^5 + 3^5 + \dots + 99^5 + 100^5$ .

7.Prove that the integer $53^{103} + 103^{53}$ is divisible by 39, and that $111^{333} + 333^{111}$ is divisible by 7.

8.Use the binary exponentiation algorithm to compute both $19^{53}$ (mod 503) and $141^{47}$ (mod 1537).

9. Find the last two digits of the number $9^{81}$.

10.Without performing the divisions, determine whether the integers 176,521,221 and 149,235,678 are divisible by 9 or 11.

11. Solve the following linear congruences:

(a) 25x = 15 (mod 29).

(b) 5x = 2 (mod 26).

(c) 6x = 15 (mod 21).

(d) 36x = 8 (mod 102).

(e) 34x = 60 (mod 98).

(f) 140x = 133 (mod 301).

12. Solve each of the following sets of simultaneous congruences:

(a) x = 1 (mod 3), x = 2 (mod 5), x = 3 (mod 7).

(b) x = 5 (mod 11), x = 14 (mod 29), x = 15 (mod 31).

13. Use Fermat's theorem to verify that 17 divides $11^{104} + 1$.

14. If gcd(a, 35) = 1, show that $a^{12} = 1$ (mod 35).
15. Find the remainder when 15! is divided by 17.
16. Find the remainder when 2(26!) is divided by 29.
17.Determine whether 17 is a prime by deciding whether 16! = -1 (mod 17).
18.show that if $F_n = 2^{2^n} + 1$, n > 1, is a prime, then 2 is not a primitive root of $F_n$.
19. 1. Find the order of the integers 2, 3, and 5:
(a) modulo 17.
(b) modulo 19.
(c) modulo 23.
20. Establish each of the statements below:
(a) If a has order hk modulo n, then $a^h$ has order k modulo n.
(b) If a has order 2k modulo the odd prime p, then $a^k = -1$ (mod p ).
(c) If a has order n- 1 modulo n, then n is a prime.
21. Prove that ¢($2^n$ - 1) is a multiple of n for any n > 1.
22. find the ¢( 6) = 2 integers having order 6 modulo 31.
23. Given that 3 is a primitive root of 43, find the following:
(a) All positive integers less than 43 having order 6 modulo 43.
(b) All positive integers less than 43 having order 21 modulo 43.
24. Find all positive integers less than 61 having order 4 modulo 61.
25. Find the four primitive roots of 26 and the eight primitive roots of 25.
26. Determine all the primitive roots of $3^2$ , $3^3$ , and $3^4$ .
27. For an odd prime p, establish the following facts:
(a) There are as many primitive roots of $2p^n$ as of $p^n$.
(b) Any primitive root r of $p^n$ is also a primitive root of p.
28. Solve the following quadratic congruences:
(a) $x^2 + 7x + 10 = 0$ (mod 11).
(b) $3x^2 + 9x + 7 = 0$ (mod 13).
(c) $5x^2 + 6x + 1 = 0$ (mod 23).
29. Prove that the quadratic congruence $6x^2 + 5x + 1 = 0$ (mod p) has a solution for every prime p, even though the equation $6x^2 + 5x + 1 = 0$ has no solution in the integers.
30. Find the value of the following Legendre symbols:
(a) (19/23).
(b) ( -23/59).
(c) (20/31).
(d) (18/43).
(e) ( -72/131).

**B.Sc-III Sem-VI**
**Skills Enhancement Course-IV**
**Python Programming U-ADC-640**

**Learning Objectives:**

1.To understand why Python is a useful scripting language for developers.

2. To learn how to install Python, start the Python shell

3. To define the structure and components of a Python program.

4. To learn to perform basic calculations, print text on the screen and perform simple control flow operations using if statements and for loops

**Course Outcomes:**

On completion of this course successfully students will be getting following skills:

- Using loops in R software
- Sorting and ordering and lists in R software

**Unit-1**

Introduction to Python and Basic Concepts in python Introduction to python: What is python? , Applications of Python, Why Python? Installation of python, First program in Python, Comments and Docstrings in Python.Variable and data types, Operators in python. File Handling : working with open, read, write, append modes of file Conditional Statements: Indentation in python, if, if-else, nested if-else statements

**Unit-2**

Looping Statements, Control statements, String Manipulations Looping Statements: for loop, while loop , Nested loops Control Statements: break, continue and pass String Manipulations: Accessing strings, Basic operations, String slices, Functions and methods

**Reference Books**

1 Introducing python - Bill Lubanovic

2. Machine Learning (in Python and R) For Dummies - John Paul Mueller

3. Core Python Programming – Dr. R.Nageswara Rao.

4. Python Cookbook - David Beazley and Brian K. Jones

**List of Practical's**

**(Student have to perform any 10 at the end of semester)**

1.Hello world program in python

2. Python Program to Check Whether a Given Year is a Leap Year

3. Python Program to Check Whether a Number is Positive or Negative

4. Print "1" if a is equal to b, print "2" if a is greater than b, otherwise print "3".Print "Hello" if a is equal to b, and c is equal to d.

5. Python Program to Read a Number n And Print the Series "1+2+…..+n= "

6. Python Program to Check if a Number is a Palindrome

7. Python Program to Count the Number of Digits in a Number

8. Python Program to Find the Sum of Digits in a Number

9. Python Program to Print Odd Numbers Within a Given Range

10. Python Program to Find the Factorial of a Number

11. Python Program to check the number is prime or not

12. Python Program to Make a Simple Calculator using function