B.Sc. I – Semester-I

Course Code- U-PHY-135

Mechanics and Properties of Matter-I

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Unit I: Gravitation (Book-1) (Periods 11)

- Introduction, Kepler's laws, Newton's law of gravitation, Newton's deductions from Kepler's laws, gravitational potential due to a spherical shell: i) at a point outside the shell, ii) at a point inside the shell, gravitational potential due to a solid sphere, numerical problems

Unit II: Elasticity (Book-1) (Periods 12)

- Introduction, twisting couple on a cylinder, torsional pendulum, determination of coefficient of modulus of rigidity of a wire: statistical method, dynamical method Maxwells needle, bending of beams, bending moment, cantilever loaded at free end: when the weight of the beam is ineffective and effective, beam loaded at the centre, numerical problems

Unit III: Surface Tension (Book-1) (Periods 11)

- Introduction, pressure difference across a liquid surface (case of drops and bubbles), rise of liquid in a capillary tube, experimental determination of surface tension by Jaeger's method and Ferguson method, factors affecting surface tension, numerical problems

Unit IV: Viscosity (Book-1) (Periods 11)

- Introduction, rate of flow of fluid, lines and tubes of flow, Reynolds number, co-efficient of viscosity, Poiseuille's equation for flow of liquid through a horizontal capillary tube, η by Poiseuille's method, Stoke's law, rotation viscometer, variation of viscosity of a liquid with temperature and pressure, numerical problems.

- Recommended Book:
- 1. Elements of Properties of Matter--- D.S Mathur, Shyamlal charitable trust, New Delhi.
- Reference Books:
- 2. General Properties of Matter---J. C. Upadhyaya, Ram Prasad and Sons publishers.
- 3. Properties of Matter --- Brijlal and Subramanyam, S. Chand and Co.

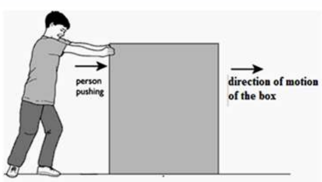
Unit I
Gravitation

GRAVITATION

THE UNIVERSAL LAW OF GRAVITATION

INTRODUCTION

- An object changes its speed or direction of motion only when a force is applied on it.



Force and Acceleration

- Application of the force accelerates the object.

Here a tennis player hits a ball and see the ball is moving very fast and its direction of motion is also changing.

WHERE DO WE ALWAYS FALL ?

Why is it that when we jump we always land on the earth, and do not remain suspended or move upwards?

What makes us to fall on the earth always?

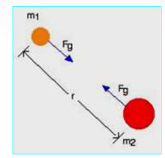
- There is always some force acting on us that guides our direction of falling.
- No matter from where ever we jump, or we drop objects from anywhere they will always fall towards the earth.

What makes the apple to always fall on the earth?

It was Isaac Newton who posed this question and answered it. Newton stated that all objects attract each other along the line joining their centers.

Every object in the universe attracts every other object towards itself

- This force with which the two objects attract each other is called the **force of gravitation**
- The force of gravitation acts even if there is nothing connecting the two objects.



The Universal Law of Gravitation

- Newton did not stop after proposing that a force of gravitation exists. He expressed the law in a clear and precise language- the language of mathematics.

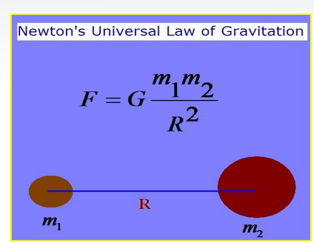
The Universal Law of Gravitation states that

- Any two point particles with masses m_1 and m_2 attract each other by a force whose magnitude is directly proportional to the product of the two masses, that is m_1m_2 and inversely proportional to the square of the distance R between them. The direction of the force is along the line joining the two masses.

The Universal Law of Gravitation states that

- Every particle of matter attracts every other particle with a force which is directly proportional to the product of their masses, and inversely proportional to the square of the distance between them.

In the mathematical form the law can be represented as…



In the expression

$$F = G\frac{m_1 m_2}{R^2}$$

G is a gravitational constant. It does not depend on the value of masses or the distance between the masses. The constant G will remain the same for any two objects anywhere in the Universe.

Value of G is 6.6734×10^{-11} Nm^2 / kg^2

We know that $F = ma$

$F = mg$ [g → Gravitational intensity of planet].

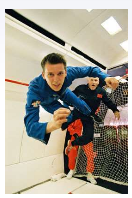
$F = \frac{GMm}{r^2}$ M → Mass of planet

$mg = \frac{GMm}{r^2}$

$\boxed{g = \frac{GM}{r^2}}$ r → Distance from centre

Uses of Gravitation

- It is the gravitational force that keep everything at its place. Otherwise we would have been floating in the air.



Weightlessness in space vehicle

It is due to gravitation that we are walking on the Earth.

Uses of Gravitation contd..

- It also keeps the earth, sun and other celestial bodies at their right places

Universal gravitation constant(G)	Acceleration due to gravity(g)
1.It is define as the force of attraction acting between two bodies each of unit mass, whose centers are placed unit distance apart.	1.It is define as the constant acceleration produced in a body when it falls freely under the effect of gravity .
2. Its value is same throughout the universe .	2. its value changes from place to palce.
3. It is a scalar quantity .	3. it is a vector quantity.
4. Its value dose not depends upon the nature and the size of the bodies.	4. Its value depends upon the nature and the size of the bodies.
5. Dimensional formula for G is $[M^{-1}L^3T^{-2}]$.	5. Dimensional formula for g is $[M^0L^1T^{-2}]$.

✓ g has Value of $9.8 m/s^2$ on Earth's surface. The value of g observed to be different from place to place because,

- Earth is not uniform.
- It is not a perfect sphere.
- It rotates.

Height in km	g in m/s^2	Place
0	9.83	On mean earth surface
8.8	9.8	On mount Everest
36.6	8.71	Highest manned balloon
400	8.7	Space shuttle orbit
35700	0.225	Communication satellite

Kepler's Three Laws of Planetary Motion

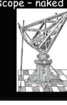
- The path of a planet is an elliptical orbit, with the sun at one of the foci.
- The radius vector, drawn from the sun to planet sweeps out areas in an equal time
- The square of a planet's year, (i.e. its time-period, or its time of revolution round the sun), is proportional to the cube of the major axis of its orbit.

Tyco Brahe

- Danish astronomer who hired Kepler as his assistant
- Came up with accurate observations of Mars with his naked eyes
- Assigned Kepler to develop a theory of planetary motion using his observations

Instruments

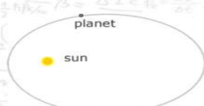
- Tyco Brahe
  - only compass and sextant
  - No telescope – naked eye



Kepler's laws:

Johannes kepler gave the following three law to explain the motion of the planets:

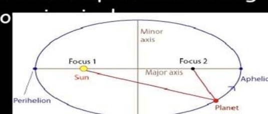
1. **Law of orbit**: Each planet moves around the sun in an elliptical orbit with the sun at one of the foci* of the orbit.



* Foci is the plural of the term focus.

Kepler's FIRST Law

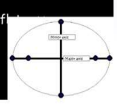
- "The orbit of each planet is an ellipse and the Sun is at one focus"
- Kepler proved Copernicus wrong – planets didn't mo



An elliptical orbit of a planet

Ellipse

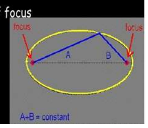
- Elongated & flattened circle
- Characterized by eccentricity and length of major axis
- Eccentricity – degree of fl
- Major axis – longer axis



KEPLER'S FIRST LAW

Focus

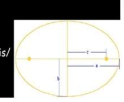
- Focus – one of two special points on the major axis of an ellipse
- Foci – plural of focus
- A+B is always the same on any point on the ellipse



KEPLER'S FIRST LAW

Eccentricity

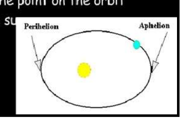
- Eccentricity is the degree of flatness
- Eccentricity (e) = 0 is circle
- Earth e = 0.017
- e = c/a
  - c = center to focus
  - a = half of major axis/ semi-major axis



KEPLER'S FIRST LAW

Aphelion & Perihelion

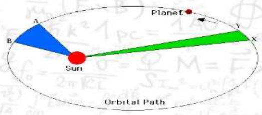
- Aphelion is the point on the orbit farthest from the sun
- Perihelion is the point on the orbit closest to the su



KEPLER'S FIRST LAW

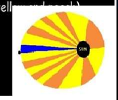
2. Law of areas : The line joining the sun and planet sweeps out equal area in equal interval of time.

let a planet moves from position A to B in time t and then it moves from position X to position Y in the same time t . Then according to kepler's second law, area SAB= area SXY



In Another Words...

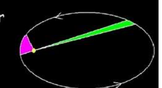
- The area from one time to another time is equal to another area with the same time interval
- All of the areas (in yell... ...) have equal intervals of time



KEPLER'S SECOND LAW

Acceleration of Planets

- Planet moves faster when closer to the sun
  - Force acting on the planet increases as distance decreases and planet accelerates in its orbit
- Planet moves slower when farther from the sun

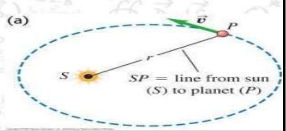


KEPLER'S SECOND LAW

3. *Law of period*: The square of time taken by a planet A to complete a revolution around the sun is directly proportional to the cube of semi-major axis of the elliptical orbit

i.e., T ∝ r or T = (constant) r^3or

$$\frac{T^2}{r^3} = constant$$



$T^2 \propto a^3$

- T = orbital period in years
- a = semi-major axis in astronomical unit (AU)
- Can calculate how long it takes (period) for planets to orbit if semi-major axis is known

KEPLER'S THIRD LAW

Astronomical Unit

- Astronomical unit - AU
- AU is the mean distance between Earth and the Sun
- 1 AU ≈ 1.5 x 10^8 km ≈ 9.3 x 10^7 miles

KEPLER'S THIRD LAW

Examples of 3rd Law

- Calculating the orbital period of 1AU
  - $T^2 = a^3$
  - $T^2 = (1)^3 = 1$
  - T = 1 year
- Calculating the orbital period of 4AU
  - $T^2 = a^3$
  - $T^2 = (4)^3 = 64$
  - T = 8 years

KEPLER'S THIRD LAW

Orbital Data

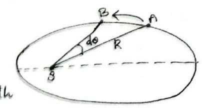
- The orbital data of various planets

Planet	eccentricity (e)	T (yr)	a (AU)	T^2	a^3
Mercury	0.206	0.24	0.39	0.06	0.06
Venus	0.007	0.62	0.72	0.39	0.37
Earth	0.017	1	1	1	1
Mars	0.093	1.88	1.52	3.53	3.51
Jupiter	0.048	11.9	5.2	142	141
Saturn	0.056	29.5	9.54	870	868

Newton's deduction from kepler's law

I. deduction

1. 1st Deduction:



Let A be the position of a planet, at a given instant "t" in its elliptical path round the sun S, situated at one of its foci. Then if the planet moves on to B in a small interval of time dt, the area swept out by the radius vector SA, in this interval of time is equal to the area of ΔSAB,

i.e. equal to $\frac{1}{2} SA \cdot AB = \frac{1}{2} R \cdot R d\theta$

$= \frac{1}{2} R^2 d\theta$

$\because SA = R,\ AB = R d\theta$.

$\therefore$ Areal velocity of planet $= \frac{1}{2} R^2 \frac{d\theta}{dt}$.

But according to kepler's second law, this must be a constant. Putting it equal to $h/2$,

we get $\frac{1}{2} R^2 \frac{d\theta}{dt} = \frac{h}{2}$

$= \frac{1}{2} R^2 d\theta$

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we get $\frac{1}{2} R^2 \frac{d\theta}{dt} = \frac{h}{2}$

$\Rightarrow R^2 \frac{d\theta}{dt} = h$.

Here the planet moves in a curved path and thus continuously changes its direction. i.e. in accordance with Newton's first law of motion, that must be under the action of force and must consequently be possessing an acceleration in the direction of force.

Resolving this acceleration into its two rectangular components, along and at right angles to the radius vector, we have

i) Component a_1, along the radius vector i.e. the radial acceleration of the planet, given by

$$a_1 = \frac{d^2R}{dt^2} - R\left[\frac{d\theta}{dt}\right]^2$$

ii) Component a_2, at right angles to the radius vector i.e. the transverse acceleration of the planet,

$$a_2 = \frac{1}{R}\frac{d}{dt}\left[R^2 \frac{d\theta}{dt}\right]$$

But we have seen that $R^2 \frac{d\theta}{dt} = h$

hence its differential coefficient is zero. $\Rightarrow a_2 = 0$

$\Rightarrow a_2 = \frac{1}{R}\frac{d}{dt}[h]$

$\boxed{a_2 = 0}$

Hence the planet has no transverse acceleration, So the only acceleration it has is the radial acceleration. and therefore the only force acting on it is towards the sun

∴ 2^{nd} Deduction :-

Now, since $R^2 \frac{d\theta}{dt} = h$

$\Rightarrow \frac{d\theta}{dt} = \frac{h}{R^2}$

Putting $\frac{1}{R} = u \Rightarrow R = \frac{1}{u}$

$\Rightarrow \frac{d\theta}{dt} = hu^2$

$\Rightarrow \frac{dR}{dt} = \frac{-1}{u^2}\frac{du}{dt} = \frac{-1}{u^2}\frac{du}{d\theta}\cdot\frac{d\theta}{dt} \quad [\because R = \frac{1}{u}]$

$\frac{dR}{dt} = -h\frac{du}{d\theta} \quad [\because \frac{d\theta}{dt} = hu^2]$

Differentiating again w.r.t t,

$\Rightarrow \frac{d^2R}{dt^2} = -h\frac{d^2u}{d\theta^2}\cdot\frac{d\theta}{dt}$

$\Rightarrow \frac{d^2R}{dt^2} = -h^2u^2\frac{d^2u}{d\theta^2} \quad [\because \frac{d\theta}{dt} = hu^2]$

Substituting the values of $\frac{d\theta}{dt}$ and $\frac{d^2R}{dt^2}$ in the expression for a_1,

As $a_1 = \frac{d^2R}{dt^2} - R\left[\frac{d\theta}{dt}\right]^2$

$\Rightarrow \quad = -h^2u^2\frac{d^2u}{d\theta^2} - R\left[hu^2\right]^2$

$= -h^2u^2\left(\frac{d^2u}{d\theta^2}\right) - \frac{1}{u}\times h^2u^4$

$= -h^2u^2\frac{d^2u}{d\theta^2} - h^2u^3$

$a_1 = -h^2u^2\left[u + \frac{d^2u}{d\theta^2}\right]$ —— (i)

Let the equation of the elliptical orbit, of the planet be

$\frac{l}{R} = 1 + e\cos\theta \Rightarrow lu = 1 + e\cos\theta$ —— (ii)

where $l \to$ latus rectum

$e \to$ eccentricity.

Differentiating this expression twice, w.r.t. θ we have

$l\frac{d^2u}{d\theta^2} = -e\cos\theta$ —— (iii)

Adding (ii) & (iii) we have

$lu + l\frac{d^2u}{d\theta^2} = 1$

$\Rightarrow l\left[u + \frac{d^2u}{d\theta^2}\right] = 1 \Rightarrow u + \frac{d^2u}{d\theta^2} = \frac{1}{l}$

$\Rightarrow l\left[u + \frac{d^2u}{d\theta^2}\right] = 1 \Rightarrow u + \frac{d^2u}{d\theta^2} = \frac{1}{l}$

Substituting this value of $\left[u + \frac{d^2u}{d\theta^2}\right]$ in relation (i)

$\Rightarrow a_1 = -h^2u^2 \times \frac{1}{l}$

$a_1 = -\frac{h^2}{l\cdot R^2} \quad [u = \frac{1}{R}]$

$\Rightarrow a_1 = -\frac{k}{R^2} \quad [\text{Putting } \frac{h^2}{l} = \text{const.} = k]$

$\Rightarrow \boxed{a_1 \propto \frac{1}{R^2}}$ ---- (iv)

i.e. the acceleration and hence the force act on the planet is inversely proportional to the square of the distance from the sun. (−ve sign indicates the force in this is the attractive force).

Now, the time-period (T) of the planet (i.e. the time taken to complete its one full revolution round the Sun) is given by

$T = \frac{\text{area of the ellipse}}{\text{areal velocity of radius vector}} = \frac{\pi ab}{\frac{1}{2}R^2\frac{d\theta}{dt}}$

where a and b are the semi-major and semi-minor axes of the elliptical orbit of the planet respectively.

$\Rightarrow T = \frac{\pi ab}{\frac{h}{2}} \quad \left[\because \frac{1}{2}R^2\frac{d\theta}{dt} = \frac{h}{2}\right]$

$$\Rightarrow T = \frac{2\pi ab}{h}$$

$$\text{Now } T^2 = \frac{4\pi^2 a^2 b^2}{h^2}$$

Now clearly $\frac{b^2}{a} = l$, the latus rectum of the ellipse and $\therefore b^2 = al$,

$$\therefore T^2 = \frac{4\pi^2 a^2 (al)}{h^2}$$

$$T^2 = \frac{4\pi^2 a^3 l}{h^2} = \frac{4\pi^2 a^3}{\left(\frac{h^2}{l}\right)}$$

$$T^2 = \frac{4\pi^2}{K} a^3 \qquad \left[\because \frac{h^2}{l} = k\right]$$

Since in accordance with kepler's third laws, $\boxed{T^2 \propto a^3}$ for every planet.

Hence $\frac{4\pi^2}{K}$ is a constant

or that K is constant for every planet.

$\therefore$ K is quite independent of nature of planet.

3rd Deduction:-

Finally if m and M are the masses of planet and the Sun and F and F' the force of attraction exerted by sun on planet and the reaction of the planet on the sun respectively, we have from relation (iv),

$$F = \frac{km}{R^2} \quad \text{and} \quad F' = \frac{KM}{R^2}$$

k and K are constants.

From Newton's Third Law of motion, action and reaction are equal and opposite,

i.e. $F = F'$

$$\Rightarrow \frac{km}{R^2} = \frac{KM}{R^2}$$

$$\Rightarrow km = KM$$

$$\Rightarrow \frac{k}{M} = \frac{K}{m} = \text{a constant say } G.$$

$$\Rightarrow k = MG \quad \text{and} \quad K = mG$$

Substituting these values k & K in F and F', we get

$$\boxed{F = G\frac{mM}{R^2}} \quad \text{and} \quad \boxed{F' = \frac{GMm}{R^2}}$$

Showing that the force of attraction between the Sun and the planet is directly proportional to the product of their masses.

Gravitational Potential:-

→ The work done in moving a unit mass from infinity to any point in the gravitational field of a body A is called the gravitational potential of that point due to body A.

→ It is usually denoted by V.

$$V = -\frac{m}{r}.G$$

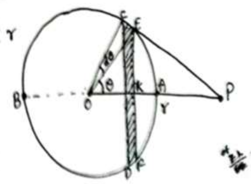
Here the value of gravitational potential at an infinite distance from a mass is zero.

It goes on decreasing as we approach the attracting mass — i.e. it is an essentially negative quantity, its maximum value being 0 at infinity where at all points, the potential will be same.

Gravitational Potential due to Spherical Shell:-

a. **At a point outside the shell:-**

Let P be the point, distant r from centre O of a spherical shell of radius R and surface density (i.e. mass per unit area of surface), σ.



Join OP and cut out a slice CEFD, in the form of ring by two plane close to each other and perpendicular to radius OA, meeting the shell in C and D and in E and F respectively. Let $\angle EOP = \theta$, small $\angle COE = d\theta$

clearly, the radius of the ring is $EK = OE\sin\theta$
$= R\sin\theta$,

So that its circumference $= 2\pi(EK)$
$= 2\pi R\sin\theta$ and its width $CE = Rd\theta$

$\therefore$ Area of the ring or slice = its circumference × width

$= 2\pi R\sin\theta \times Rd\theta$
$= 2\pi R^2 \sin\theta d\theta$

Then its mass $= 2\pi R^2 \sin\theta\, d\theta \times \sigma$ $\quad \left[\because \frac{m}{A} = \sigma\right]$

If $EP = x$, every point of the slice is at a distance x from P and therefore, the potential at P due to this small ring/slice is given by

$$dV = \frac{-\text{mass of slice}}{x} \cdot G = -\frac{2\pi R^2 \sin\theta\, d\theta\, \sigma}{x} \cdot G \quad \text{—(i)}$$

Now in Δ^{le} OEP,

$$EP^2 = OE^2 + OP^2 - 2OE \cdot OP \cdot \cos\theta$$

$$x^2 = R^2 + r^2 - 2Rr\cos\theta \qquad \because OE = R,\ OP = r.$$

$$\Rightarrow 2x\,dx = 0 + 0 + 2Rr\sin\theta\, d\theta$$

$$\Rightarrow x = \frac{\cancel{2}Rr\sin\theta\, d\theta}{\cancel{2}dx}$$

Sub. in (i).

$$dV = \frac{-2\pi R^{\cancel{2}} \sin\theta\, d\theta\, \sigma}{\cancel{R} r \sin\theta\, d\theta} \times dx \cdot G$$

$$dV = -\frac{2\pi R\sigma dx}{r} \cdot G = -\frac{2\pi R\sigma G}{r} \cdot dx$$

Integrating this between the limits $x = AP = (r-R)$ to $x = BP = (r+R)$, we get V, the potential due to the whole shell at the point P.

Thus $V = \int_{r-R}^{r+R} -\frac{2\pi R\sigma G}{r}\,dx$

$= \frac{-2\pi R\sigma G}{r}\int_{(r-R)}^{(r+R)} dx$

$= \frac{-2\pi R\sigma G}{r}\left[x\right]_{(r-R)}^{(r+R)} = \frac{-2\pi R\sigma G}{r}\left[r+R-(r-R)\right]$

$V = \frac{-2\pi R\sigma G}{r}(2R) = -\frac{4\pi R^2\sigma G}{r}$

Now $4\pi R^2$ is the surface area of the whole shell,

$\therefore$ $(4\pi R^2\sigma)$ is equal to its mass M.

Thus we have

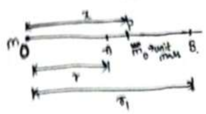
$$V = -\frac{M}{r}G$$

or, the potential at the point P due to the whole shell is equal to $-\frac{M \cdot G}{r}$, i.e. the same as it would be due to a mass M at O.

The mass of the whole shell thus behaves as though it were concentrated at its centre.

Gravitational potential at a point distant r from a body of mass m :-

Let a mass m be situated at O and let a unit mass be situated at P. Then the force of attraction of the unit mass due to m is clearly,



$= \frac{m \times 1}{x^2}\cdot G = \frac{m}{x^2}\cdot G$

where x is the distance between O & P from O the force being directed towards O.

Therefore, work done when unit mass moves through a distance dx towards O, is equal to

$\frac{m \cdot G}{x^2} \times dx$.

And therefore, work done when it moves from B to A

$= \int_B^A \frac{m}{x^2} G\,dx = Gm\int_B^A \frac{1}{x^2}dx$

$= Gm\left[-\frac{1}{x}\right]_B^A$

$= -Gm\left[\frac{1}{x}\right]_{r_1}^{r}$

$= -Gm\left[\frac{1}{r} - \frac{1}{r_1}\right] = Gm\left[\frac{1}{r_1} - \frac{1}{r}\right]$

where r and r_1 are the distances of A and B from O. That is, is the potential difference between the points A and B.

If B is infinity, i.e. $r_1 = \infty$, we have

Potential difference between A & ∞ $= Gm\left[\frac{1}{\infty} - \frac{1}{r}\right]$

$V = \frac{-m}{r}G$

$\therefore$ Potential difference between A and ∞ is equal to the potential at A. [Because Gravi. force at ∞ $= 0 \Rightarrow$ work done at $\infty = 0$ $\Rightarrow V_{at\,\infty} = 0$]

Gravitational potential at point distant r from body mass m by is V

$$\boxed{V = \frac{-m}{r}\cdot G.}$$

b) At a point on the surface of the shell:-

In the case i, if we imagine the point P to lie at A, i.e. on the surface of the shell, itself. We obtain the potential there by integrating the expression for dV between the limits $x=0$ to $x=2R$. So that in this case, i.e. $r = R$, $\therefore x = r-R = R-R = 0$, $x = r+R = R+R = 2R$.

$V = \int_0^{2R} \frac{2\pi R\rho G}{r}\cdot dx$

$= -\frac{2\pi R\rho G}{r}\left[x\right]_0^{2R} = \frac{-2\pi R\rho G(2R)}{r}$

$V = -\frac{4\pi R^2\rho G}{r}$

$\Rightarrow \boxed{V = -\frac{MG}{R}}$ [$\because r = R$ on the surface)

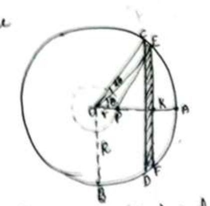
Again therefore, the whole mass of the shell behaves as though it were concentrated at its centre.

c) At a point inside the shell:-

Now the point lies inside the spherical shell. Then repeating the case 1 procedure,

potential at P due to the slice or ring CEFD, i.e.

$$dV = \frac{2\pi R \rho G}{r}\, dx$$



In this case, the limits of x are $(R-r)$ and $(R+r)$ $[\because R > r)$

So that we have

$$V = \int_{R-r}^{R+r} -\frac{2\pi R \rho G}{r}\, dx$$

$$= -\frac{2\pi R \rho G}{r}\,[dx]_{R-r}^{R+r}$$

$$= -\frac{2\pi R \rho G}{r}\,[2r]$$

$$V = -4\pi R \rho G$$

Multiplying and dividing by R, we have

$$V = -\frac{4\pi R^2 \rho G}{R}$$

$$V = -\frac{MG}{R} \qquad \because 4\pi R^2 \rho = M = \text{mass of shell}.$$

Hence $\boxed{V = -\frac{MG}{R}}$

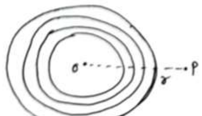
i.e. the same as the point on the shell.

Hence the above value of V has been obtained for a point P anywhere inside the shell, it follows that the potential at all the points inside the spherical shell is same and numerically equal to the value of the potential on the surface of the shell itself, which is also its maximum (negative) value.

Gravitational Potential due to Solid Sphere:-

a. At a point outside the sphere:-

Let the point P lie outside a sphere of radius R mass M at a distance r from the centre, O. Imagine the sphere to consist of a large number of thin shells, concentric with the shells sphere, one inside the other, as shown and let their respective masses be $m_1, m_2, m_3, \ldots$ etc.



→ We know that, the potential at P due to each spherical shell will be equal to its mass $\times \frac{G}{r}$

$= M \times \frac{G}{r}$, i.e.

→ We know that, the potential at P due to each spherical shell will be equal to its mass $\times \frac{G}{r}$

$= M \times \frac{G}{r}$, i.e.

Same as though the mass of each shell is concentrated at the centre.

→ So that the potential at P due to different shells will be $-m_1 \cdot \frac{G}{r}, -\frac{m_2 G}{r}, -m_3 \frac{G}{r}, \ldots$

∴ Potential at P due to all the shells, i.e. due to the whole solid sphere i.e.

$$V = -\left[\frac{m_1 \cdot G}{r} + \frac{m_2 \cdot G}{r} + \frac{m_3 \cdot G}{r} + \cdots\right]$$

$$V = -(m_1 + m_2 + m_3 + \cdots)\cdot\frac{G}{r} = -\frac{M}{r}\cdot G$$

because $m_1 + m_2 + m_3 + \cdots = M$, mass of the solid sphere.

Thus, {potential due to solid sphere $= -\frac{MG}{r}$}

b) At a point on the surface of the sphere:-

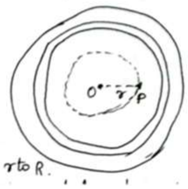
If $r = R$, we have

Potential at the surface of the sphere $= -\dfrac{M \cdot G}{R}$.

c) At a point inside the sphere:-

Let the point P now lie inside the solid sphere of radius R, mass M and volume density ρ at a distance r from its centre O.

The solid sphere may be imagined to be made up of an inner solid sphere of radius r, surrounded by a number of hollow spheres or spherical shells, concentric with it and with their radii ranging from r to R.



The potential at P due to the solid sphere is, therefore equal to the sum of the potentials at P due to the inner solid sphere and all such spherical shells outside it.

Clearly P lies on the surface of the solid sphere of radius r and inside all the shells of radii greater than r.

$\therefore$ Potential at P due to the inner solid sphere of radius $r = -\dfrac{\text{mass of sphere}}{r} \times G = -\dfrac{\frac{4}{3}\pi r^3 \rho G}{r}$

$= -\frac{4}{3}\pi r^2 \rho G \quad \left[\because \text{mass of sphere} = \frac{4}{3}\pi r^3 \cdot \rho\right]$

To determine the potential at P due to the outer shells, consider a shell of radius x and thickness dx, so that its volume = area × thickness $= 4\pi x^2 dx$ and its mass $= 4\pi x^2 dx \cdot \rho$

Now, the potential at any point within a shell is the same as at any point on ~~the~~ its surface and therefore potential at P due to this shell

$$= -\frac{4\pi x^2 dx \cdot \rho \cdot G}{x} = -4\pi x \, dx \, \rho G.$$

Integrating this for the limits $x = r$ to $x = R$, we get the potential at P due to all the shells. Thus,

Potential at P due to all the shells

$$= -\int_r^R 4\pi \rho G \cdot x \, dx$$

$$= -4\pi \rho G \int_r^R x \, dx$$

$$= -4\pi \rho G \left[\frac{x^2}{2}\right]_r^R$$

$$= -\frac{4}{2}\pi \rho G \left[R^2 - r^2\right]$$

$$= -\frac{4}{3}\pi \rho G \, 3\left[\frac{R^2 - r^2}{2}\right]$$

$\therefore$ Total potential at $P = -\frac{4}{3}\pi r^2 \rho G - \frac{4}{3}\pi \rho G \left(\frac{3R^2 - 3r^2}{2}\right)$

$$= -\frac{4}{3}\pi \rho G \left[r^2 + \frac{3R^2}{2} - \frac{3r^2}{2}\right]$$

$$= -\frac{4}{3}\pi \rho G \left[\frac{3R^2 - r^2}{2}\right]$$

$$= -\frac{4}{3}\pi R^3 \rho G \left[\frac{3R^2 - r^2}{2R^3}\right]$$

Potential at $P\,(V) = -M \cdot G \left[\frac{3R^2 - r^2}{2R^3}\right]$ $\quad \left[\because \frac{4}{3}\pi R^3 \rho = M\right]$

(mass of the solid sphere)

If $r = 0$, i.e. Point P would lie at the centre of the sphere and we shall have the potential at the centre of the sphere

$$V_{centre} = -M \cdot G \cdot \frac{3R^2}{2R^3}$$

$$= \frac{-3MG}{2R}$$

Hence Potential at centre : Potential on surface $= V_{centre} : V_{surface}$

$$= -\frac{3MG}{2R} : -\frac{MG}{R}$$

$$= 3 : 2$$