

Impact Factor – 6.625

E-ISSN – 2348-7143

INTERNATIONAL RESEARCH FELLOWS ASSOCIATION'S  
**RESEARCH JOURNEY**

International Multidisciplinary E-Research Journal

Peer Reviewed & Indexed Journal

July 2020 Special Issue -250



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Assist. Prof. (Marathi)

MGV's Arts, Science & Commerce College,

Harsul, Tal. Tryambakeshwar

Dist – Nashik [M.S.] INDIA

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'RESEARCH JOURNEY' International Multidisciplinary E- Research Journal

Impact Factor - (SJIF) - 6.625 (2019)

Special Issue 250

Peer Reviewed & Indexed Journal

E-ISSN :

2348-7143

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## INDEX

No.	Title of the Paper	Author's Name	Page No.
1	The Supernatural Entanglement in S. T. Coleridge's 'Christabel'	Dr. Leena Pandhare	05
2	Adoption of Technology in Education - A Significant Task	Dr. Jitendra Shinde	08
3	Role and Types of Non-Banking Financial Companies (NBFCs)	Dr. Sanjay Argade	11
4	Cyber Security in IoT (Internet of Things) and IIoT (Industrial Internet of Things)	Sandesh Jadhav	19
5	Gandhian Views on Health	P.V.Watkar	27
6	Important Things to be Considered before and while Investing in Different Investment Plans	Dr. Sanjay Argade	32
7	Population Resource Perspective in Tribal Area of Salekasa Taluka, Gondia District, Maharashtra	Dr. Jyoti Rokde, Dr. Rajani Chaturvedi, Mr. Ravindra Patil	39
8	Cultural Blending of Indo-Parsi Traditions : An Analysis	Sumedha Dwivedi	53
9	Noise Pollution in Shivjayanti in Pune City (2014)	Dr. Pandurang Patil	59
10	Multiculturalism in Kiran Desai's <i>The Inheritance of Loss</i>	Dr. S. C. Vyawahare	62
11	Exploring Identity Through Myth in Bharti Kirchner's Shiva Dancing	Tulika Ghosh	67
12	Practical Approach to Creation of Digital Libraries	Dr. B. V. Chalukya	72
13	Noise Pollution of Salience Zone of P.M.C. and P.C.M.C. of Pune City (2010-2011)	Dr. Pandurang Patil	81
14	Spatial Analysis of Tribal Handicrafts Population in Nandurbar District (Maharashtra)	Mr. Mohan Vasave, Dr. U. V. Nile	85
15	Topological Methods for Solutions of Boundary Value Problems	S. B. Birajdar, S. P. Birajdar, N. S. Pimple	90
16	Stability of Difference Equations	Dr. G. B. Lamb	95
17	Non-Local Cauchy Problem for Summation Difference Equations	Dr. S. R. Gadhe	99
18	Application of Group Theory To Chemical Bonding	Dr. D. D.Kadam	103
19	Some Solutions of Difference Equations and Difference Inequalities in Fixed Point Theory	Dr. G.B. Lamb	110
<b>हिंदी विभाग</b>			
20	शिवानी के उपन्यासों में समसामायिक बोध	डॉ. संजय ढोडरे, प्रा. अंजीर भील	115
21	महाविद्यालयीन स्तर के शहरी एवं ग्रामीण छात्राओं में खेलों के प्रति रुचि में कमी के कारणों का तुलनात्मक अध्ययन	डॉ. अनिता कोल्हे	118
22	हरिकृष्ण प्रेमी के नाटकों में राष्ट्रीय चेतना	डॉ. सचिन कुमावत	121
23	'धुली हुई शाम' कहानी संग्रह में नारी चेतना	डॉ. संजय ढोडरे	124
24	वाल्मीकि रामायण में लोकमत की अवधारणा एवं प्रभाव	अंजू रानी	127
25	शशिप्रभा शास्त्री कृत उपन्यास 'परछाइयों के पीछे' : एक अनुशीलन	डॉ. संजय ढोडरे	131
<b>मराठी विभाग</b>			
26	एकविसावे शतक आणि मराठी साहित्यासमोरील आव्हाने	डॉ. पृथ्वीराज तौर	135
27	मच्छीमार समाजाच्या 'सात देवी'	डॉ. अंजली मस्करेन्स	147
28	महानुभावीय वाङ्मयातून येणाऱ्या ग्रामीण जीवन जाणीवा	डॉ. मारोती घुगे	152
29	धुळे व नंदुरबार जिल्ह्यातील मावची बोलीतील लोकोत्सव	महेंद्र गावित, डॉ. वासुदेव बले	157
31	निमाडी व मालवी भाषेचा बंजारा बोलीशी तुलनात्मक अभ्यास	डॉ. व्ही. बी. राठोड	162



### **5. Conclusion:**

Nandurbar district has a large tribal population. Nandurbar district consists of Akkalkuwa, Dhadgaon, Taloda, Shahada, Nandurbar and Navapur talukas. Akkalkuwa, Dhadgaon, Taloda and Shahada talukas are included in the western Satpuda hills. This mountainous region is a remote and densely forested region with a large tribal population. Because the spread of education is low, a family of 10 to 12 people can be seen. Nandurbar and Navapur talukas are part of the Western Sahyadri hills. The spread of education in this area is good and the population is small. Usually a family consists of 4 to 5 people. According to the 2001 to 2011 census, the population of Nandurbar district has increased.

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## Topological Methods for Solutions of Boundary Value Problems

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### Abstract:

*In present paper we have proved the existence of solutions of boundary value problems associated with the three dimensional system using topological methods.*

**Keywords:** BVP, Existence Theorem, Egress point, Retraction Mapping, Wazewski's topological method.

### 1.Introduction

A variety of techniques are employed in the theory of finding solution of boundary value problems. In this paper various topological principles are utilized in solving boundary value problems. The Wazewski's topological method together with the connected properties of solution Funnels is used to prove the existence of solutions.

#### 1.1 Solution Funnels

Consider the existence of solution of boundary value problems associated with the three dimensional system

$$x' = f(t, x, y, z), y' = g(t, x, y, z), z' = h(t, x, y, z) \quad \text{--- (1.1)}$$

The following preliminary notation will be needed

Let  $\Psi(t, y, z)$  and  $\Phi(t, y, z)$  be continuous functions such that

$$\Psi(t, y, z) \geq \Phi(t, y, z) \text{ for } t \geq 0, |y| < \infty, |z| < \infty.$$

Define the following sets:

$$Q(t_0) = [(t, x, y, z) : 0 \leq t \leq t_0; |x| + |y| + |z| < \infty],$$

$$Q(\infty) = [(t, x, y, z) : 0 \leq t < \infty; |x| + |y| + |z| < \infty],$$

$$R = [(t, x, y, z) : 0 \leq t < \infty; |y| < \infty, \Phi(t, y, z) \leq x \leq \Psi(t, y, z)],$$

$$C(t) = [(x, y, z) : (t, x, y, z) \in R],$$

$$S_{\Psi}(t) = [(x, y, z) : (x, y, z) \in C(t), x \in \Psi(t, y, z)],$$

$$S_{\Phi}(t) = [(x, y, z) : (x, y, z) \in C(t), x \in \Phi(t, y, z)]$$

Let  $f(t, x, y, z)$  and  $g(t, x, y, z)$  be continuous in a set  $E$  which is open relative to  $Q(t_0)$  where  $Q(t_0) \cap \mathbb{R} \subset E$ . For  $t_1, t_2 \geq 0$  and let  $S$  be a subset of  $C(t_1)$  and denote by  $I_E(S, t_1,$



$\hat{t}_2$ ) the set of all points  $(x, y)$  such that there is a solution of  $(x(\hat{t}), y(\hat{t}), z(\hat{t}))$  of (1.1) on  $(\hat{t}_1, \hat{t}_2)$  in which  $(x(\hat{t}_1), y(\hat{t}_1), z(\hat{t}_1)) \in S$  also  $(x(\hat{t}_2), y(\hat{t}_2), z(\hat{t}_2)) = (x, y, z)$  and  $((x(\hat{t}), y(\hat{t}), z(\hat{t})) \in E$  for all  $t \in [\hat{t}_1, \hat{t}_2]$ . This set is called the solution Funnel cross section

at  $t = \hat{t}_2$ .

Before stating our main result, we shall need the following hypotheses which restrict the behaviour of solutions as they cross the  $\Psi$  and  $\Phi$  surfaces.

We shall assume:

**(H<sub>1</sub>)** For all  $\hat{t}_1 \in [0, \hat{t}_0]$ ,  $(x_1, y_1, z_1) \in I_E(S_1, 0, \hat{t}_1) \cap S_{\Psi(\hat{t}_1)}$  implies that there exists a solution of (1.1) emanating from  $(\hat{t}, x_1, y_1, z_1)$  with trajectory which is on or above the  $\Psi$  surface, on some right neighbourhood of  $\hat{t}_1$ .

**(H<sub>2</sub>)** For all  $\hat{t}_1 \in [0, \hat{t}_0]$ ,  $(x_1, y_1, z_1) \in I_E(S_1, 0, \hat{t}_1) \cap S_{\Phi(\hat{t}_1)}$  implies that there exists a solution of (1.1) emanating from  $(\hat{t}, x_1, y_1, z_1)$  with trajectory which is on or below the  $\Phi$  surface, on some right neighbourhood of  $\hat{t}_1$ .

## 2. Main Results

Our first result describes qualitatively the behaviour of solutions of (1.1) assuming **(H<sub>1</sub>)** and **(H<sub>2</sub>)** holds.

### Theorem 2.1

Let  $S_1$  be a compact connected set in  $C(0)$  which intersects  $S_{\Psi(0)}$  and  $S_{\Phi(0)}$ . Then

- (i)  $I_E(S_1, 0, t)$  contains a compact connected component in  $C(t)$  which intersects both  $S_{\Psi(t)}$  and  $S_{\Phi(t)}$  for all  $\hat{t}_1 \in [0, \hat{t}_0]$ , or
- (ii) **There is** a solution of (1.1) with  $(x(0), y(0), z(0)) \in S_1$  having a maximal interval of  $[0, t^+] \subset [0, \hat{t}_0]$  such that  $|y(t)| \rightarrow \infty, |z(t)| \rightarrow \infty$  at  $t \rightarrow t^+$ .

**Proof:** Let  $E' = \{(t, x, y, z) \in E, \Phi(t, y, z) - 1 < x < \Psi(t, y, z) + 1\}$ . Assume there is no number  $K$  such that for all  $t \in [0, \hat{t}_0]$ ,  $0, t)$  implies  $|x| \leq K, |y| \leq K,$

$|z| \leq K$ . Then there is some interval  $[0, t^+] \subset [0, \hat{t}_0]$  such that for any  $K > 0$  there exists an  $\epsilon(K) > 0$  such that either  $|x| > K$ , or  $|y| > K$ , or  $|z| > K$  for  $t \in [t^+ - \epsilon, t^+]$ .

**Take any sequence**  $\hat{t}_n \rightarrow t^+$  and let  $(x_n, y_n, z_n) \in I_{E'}(S_1, 0, \hat{t}_n)$ . Then either  $|x_n| \rightarrow \infty$  or  $|y_n| \rightarrow \infty$  or  $|z_n| \rightarrow \infty$ . By the continuity of  $\Psi(t, y, z)$  and  $\Phi(t, y, z)$  it follows that if  $|x_n| \rightarrow \infty$  then  $|y_n| \rightarrow \infty, |z_n| \rightarrow \infty$ . Hence either case we have  $|y_n| \rightarrow \infty, |z_n| \rightarrow \infty$ .

**By a standard diagonalization process, using the solutions associated**  $\{(x_n, y_n, z_n)\}$ , we may construct a solutions  $(x(\hat{t}), y(\hat{t}), z(\hat{t}))$  of (1.1) with  $(x(0), y(0), z(0)) \in S_1$  which exists on  $[0, t] \subset [0, t^+]$  and such that  $|y(\hat{t})| \rightarrow \infty, |z(\hat{t})| \rightarrow \infty$  as  $t \rightarrow t^+$ .

Moreover,  $(t, x(t), y(t), z(t)) \in E' \subset E$ . This is the case (ii), hence to complete the proof we may assume there exists a number  $K > 0$ , such that for all  $t \in (0, \hat{t}_0]$ ,

$(x, y, z) \in I_{E'}(S_1, 0, t)$  implies  $|x| \leq K, |y| \leq K, |z| \leq K$ .

Let  $T$  be the set of all points  $\bar{t} \in [0, \hat{t}_0]$  such that for all  $t \in [0, \bar{t}]$ , the set  $I_{E'}(S_1, 0, t)$  contains a component in  $C(t)$  which intersects both  $S_{\Psi(t)}$  and  $S_{\Phi(t)}$ . Then  $T$  is nonempty since  $0 \in T$



and is bounded above by  $t_0$ . We shall show  $t$  is closed and thus if we let

$s = \text{Sup } T$ , then  $s = t_0$  implies the conclusion of theorem (1.1).

Let  $\{s_i\}$  be a sequence of points in  $T$  converging to  $s$  with  $C_i \subset C(s_i)$  a component of  $I_{E'}(S_1, 0, s_i)$  which intersects both  $S_{\Psi(s)}$  and  $S_{\Phi(s)}$ . Let  $L$  denote the limit set of  $\{C_i\}$  and let  $(a, b, c) \in L$ , since  $L$  is nonempty. Then there exists a sequence  $\{(x'_i, y'_i, z'_i)\}$  converging to  $(a, b, c)$  where  $(x'_i, y'_i, z'_i) \in C'_i$  and  $\{C'_i\}$  is a subsequence of  $\{C_i\}$ .

Let  $L'$  be the set of limit points of  $\{C'_i\}$ ; hence  $L'$  is compact and since  $C'_i$  intersects both  $S_{\Psi(s)}$  and  $S_{\Phi(s)}$  for all  $i$  then  $L'$  intersects both  $S_{\Psi(s)}$  and  $S_{\Phi(s)}$ .

If  $L'$  contains no component which intersects both  $S_{\Psi(s)}$  and  $S_{\Phi(s)}$ , then since  $L'$  is compact and intersects both  $S_{\Psi(s)}$  and  $S_{\Phi(s)}$ ,  $L'$  is the union of two non-empty compact sets  $M$  and  $N$  which are separated by an arc  $A$  such that  $A \cap L' = \emptyset$ .

Assume  $(a, b, c) \in M$  and  $(c, d, e)$  be any point in  $N$  and let  $(\bar{x}_i, \bar{y}_i, \bar{z}_i) \rightarrow (c, d, e)$ , where  $(\bar{x}_i, \bar{y}_i, \bar{z}_i) \in C''_i$  a subsequence of  $\{C'_i\}$ . Choose  $\{(x''_i, y''_i, z''_i)\}$  to be the subsequence of  $\{(x'_i, y'_i, z'_i)\}$  contained in  $C''_i$ . For  $i$  sufficiently large the connected set  $I_{E'}(C''_i, s''_i, s)$  intersect  $A$  and let  $(p_i, q_i, r_i)$  be the points of intersection.

Thus  $(p_i, q_i, r_i) \in I_{E'}(C''_i, s''_i, s) \cap A$ .

There exists a point  $(p, q, r)$  which is limit point of  $\{(p_i, q_i, r_i)\}$  and since  $s''_i \rightarrow s$ ,  $(p, q, r)$  is a limit point of  $\{C''_i\}$ . However then  $(p, q, r) \notin L'$  and  $(p, q, r) \in A \cap L'$ . This is a contradiction and we conclude  $L'$  and thus  $I_{E'}(S_1, 0, s)$  contains a component  $c$  in  $C(S)$  intersection both  $S_{\Phi(s)}$  and  $S_{\Psi(s)}$ . Thus  $T$  is closed.

By assumptions  $(H_1)$  and  $(H_2)$  there exists a  $\delta > 0$  such that

$I_{E'}(C, s, t) \subset \{(x, y, z) : (t, x, y, z) \in E'\}$  and  $I_{E'}(C, s, t)$  contains a component which intersects both  $S_{\Phi(t)}$  and  $S_{\Psi(t)}$  for all  $t \in (s, s + \delta)$ .

However this contradicts the fact that  $s = \text{Sup } T$ . Thus  $s = t_0$  and (i) is proved. This completes the proof of theorem.

Theorem 2.1 with conditions that restrict the possibility of (ii) occurring can be used to deduce existence theorems.

The following condition will thus be imposed:

**(H<sub>3</sub>)** Given any  $n > 0$  and  $t_0 > 0$  there exists a number  $N(t_0, n)$  such that for any solution  $(x(t), y(t), z(t))$  of (1.1) with  $|y(0)| < n_1$ ,  $|z(0)| < n_2$  and  $(t, x(t), y(t), z(t)) \in E$  for  $t \in [0, t_0]$  we have  $|y(t)| < N(t_0, n_1)$ ,  $|z(t)| < N(t_0, n_2)$ , for all  $t \in [0, t_0]$ .

### Theorem 2.2 (Existence Theorem)

Assume conditions  $(H_1)$  -  $(H_3)$  hold. Let  $S_1$  be a compact connected in  $C(0)$  which intersects both  $S_{\Psi(0)}$  and  $S_{\Phi(0)}$ . Let  $S_2$  be a closed connected subset of  $C(t_0)$  such that

$S_2 \cap \{(x, y) : y \text{ is arbitrary}\} \neq \emptyset$ , then there exists a solution of  $(x(t), y(t), z(t))$  of (1.1) on  $[0, t_0]$  such that  $(x(0), y(0), z(0)) \in S_1$ ,  $(x(t_0), y(t_0), z(t_0)) \in S_2$  with

$(t, x(t), y(t), z(t)) \in E$  for  $t \in [0, t_0]$ .





**Proof:** Since  $S_1$  is a compact set, let  $n_1 = \text{Sup}|y|$ ,  $n_2 = \text{Sup}|z|$  for  $(x, y, z) \in S_1$ , and let  $N = N(t_0, n)$  be as in  $(H_3)$ . Then  $|y(t)| < N(t_0, n_1)$  for any  $(x(t), y(t), z(t))$  of (1.1) with  $(x(0), y(0), z(0)) \in S_1$  and all  $t \in [0, t_0]$ . By theorem 2.1,  $I_E(S_1, 0, t_0)$  contains a compact connected component  $c$  to  $C(t_0)$  which intersects both  $S_{\Psi(t_0)}$  and  $S_{\Phi(t_0)}$ . The conditions imposed on  $S_2$  insure that  $S_2 \cap I_E(S_1, 0, t_0) \neq \emptyset$ . This concludes the proof of the theorem.

Another approach to this problem is the application of Wazewski's topological method.

Consider the differential system

$$\left. \begin{aligned} x' &= f(t, x, y), & x(t_0) &= x_0, & t_0 > 0 \\ y' &= f(t, x, y), & y(t_0) &= y_0, & t_0 \geq 0 \end{aligned} \right\} \text{--- (2.1)}$$

Where  $f \in C[\Omega, R^n]$ ,  $\Omega$  being any open set in  $R^{n+1}$ . Let  $\Omega_0$  be an open set of  $\Omega$ ,  $\partial\Omega_0$  is the boundary and  $\overline{\Omega_0}$  the closure of  $\Omega_0$ .

### Definition 2.1

A point  $(t_0, x_0) \in \Omega \cap \partial\Omega_0$  and  $(t_0, y_0) \in \Omega \cap \partial\Omega_0$  is said to be egress point of  $\Omega_0$  with respect to the system (2.1) if, for every solution  $x(t), y(t)$  of (2.1), there is an  $\epsilon > 0$  such that  $(t, x(t)) \in \Omega_0, (t, y(t)) \in \Omega_0$  for  $t_0 - \epsilon \leq t < t_0$ .

An egress point  $(t_0, x_0)$  and  $(t_0, y_0)$  of  $\Omega_0$  is called a strict egress point of  $\Omega_0$ ,

if  $(t, x(t)) \notin \overline{\Omega_0}$  and  $(t, y(t)) \notin \overline{\Omega_0}$  for  $t_0 < t \leq t_0 + \epsilon$  for a  $\epsilon > 0$ .

Denote the set of all points of egress (strict egress) as  $\mathbb{S} (\mathbb{S}^*)$ .

### Definition 2.2

If  $A \subset B$  are any two sets of a topological space  $X$  and  $\pi : B \rightarrow A$  is a continuous mapping from  $B$  into  $A$  such that  $\pi(p) = p$  for every  $p \in A$ , then  $\pi$  is said to be a retraction of  $B$  onto  $A$ .

When there exists a retraction of  $B$  onto  $A$ ,  $A$  is called retract of  $B$ .

### Theorem 2.3

Let  $f \in C(\Omega, R^n)$ ,  $\Omega$  open in  $R^{n+1}$ . Assume that through every point of  $\Omega$  there passes a unique solution of (2.1). Let  $\Omega_0$  be an open subset of  $\Omega$ . Suppose that all egress points of  $\Omega_0$  are strict egress points. Let  $U$  be a nonempty subset of  $\Omega_0 \cup \mathbb{S}$  such that  $U \cap \mathbb{S}$  is a retract of  $\mathbb{S}$ , but not a retract of  $U$ . Then there exists at least one point  $(t_0, x_0, y_0) \in U \cap \Omega_0$  such that the solution  $(t, x(t), y(t))$  of (2.1) remains in  $\Omega_0$  on its maximal interval of existence to the right of  $t_0$ .

We now apply theorem 2.3 to BVP prescribed in theorem 2.2. Although we assume uniqueness of solutions of (1.1) this is not essential as a Wazewski-like theorem for non-uniqueness has been developed. Let  $\Omega = [(t, x, y, z) : t \geq 0] - S_2$ ; then  $\Omega$  is a relatively the half space open subset of  $[(t, x, y, z) : t \geq 0]$ .

Let  $\Omega_0 = [(t, x, y, z) : 0 \leq t < t_1, \Phi(t) < x < \Psi(t), |y| < \infty, |z| < \infty]$  where now  $\Psi(t, y, z), \Phi(t, y, z)$  are independent of  $y$  and  $z$ . Let  $U = S_1$ .

From hypothesis  $(H_1)$  and  $(H_2)$ , it is not difficult to see that  $\mathbb{S} = \mathbb{S}^*$ , and  $\mathbb{S}$  consists of the union of the sets