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INDEX

5. Conclusion:

Nandurbar district has a large tribal population. Nandurbar district consists of Akkalkuwa, Dhadgaon, Taloda, Shahada, Nandurbar and Navapur talukas. Akkalkuwa, Dhadgaon, Taloda and Shahada talukas are included in the western Satpuda hills. This mountainous region is a remote and densely forested region with a large tribal population. Because the spread of education is low, a family of 10 to 12 people can be seen. Nandurbar and Navapur talukas are part of the Western Sahyadri hills. The spread of education in this area is good and the population is small. Usually a family consists of 4 to 5 people. According to the 2001 to 2011 census, the population of Nandurbar district has increased.

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Topological Methods for Solutions of Boundary Value Problems

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Abstract:

In present paper we have proved the existence of solutions of boundary value problems associated with the three dimensional system using topological methods.

Keywords: BVP, Existence Theorem, Egress point, Retraction Mapping, Wazewski's topological method.

1.Introduction

A variety of techniques are employed in the theory of finding solution of boundary value problems. In this paper various topological principles are utilized in solving boundary value problems. The Wazewski's topological method together with the connected properties of solution Funnels is used to prove the existence of solutions.
solution Funnels is used to prove the existence of solutions.

1.1 Solution Funnels

Consider the existence of solution of boundary value problems associated with the three dimensional system

$x' = f(t, x, y, z), y' = g(t, x, y, z), z' = f(t, x, y, z)$ --- (1.1)

The following preliminary notation will be needed

Let $\Psi(t, y, z)$ and $\Phi(t, y, z)$ be continuous functions such that

$\Psi(t, y, z) \geq \Phi(t, y, z)$ for $t \geq 0$, $|y|<\infty$, $|z|<\infty$.

Define the following sets:

$Q(t_0) = [(t, x, y, z) : 0 \leq t \leq t_0; |x| + |y| + |z| < \infty]$,

$Q(\infty) = [(t, x, y, z) : 0 \leq t < \infty; |x| + |y| + |z| < \infty]$,

$R = [(t, x, y) : 0 \leq t < \infty; |y| < \infty, \Phi(t, y, z) \leq x \leq \Psi(t, y, z)]$,

$C(t) = [(x, y) : (t, x, y, z) \in \mathbb{R}]$,

$S_{\Psi(t)} = [(x, y, z) : (x, y, z) \in C(t), x \in \Psi(t, y, z)]$,

$S_{\Phi(t)} = [(x, y, z) : (x, y, z) \in C(t), x \in \Phi(t, y, z)]$

Let $f(t, x, y, z)$ and $g(t, x, y, z)$ be continuous in a set E which is open relative to $Q(t_0)$ where $Q(t_0) \cap \mathbb{R} \subset E$. For $t_1, t_2 \geq 0$ and let S be a subset of $C(t_1)$ and denote by $I_E(S, t_1,$

t_2) the set of all points (x, y) such that there is a solution of (x(t), y(t), z(t)) of (1.1) on (t_1, t_2) in which (x(t_1), y(t_1), z(t_1)) ∈ S also (x(t_2), y(t_2), z(t_2)) = (x, y, z) and ((x(t), y(t), z(t)) ∈ E for all t ∈ [t_1, t_2]. This set is called the solution Funnel cross section

at t = t_2.

Before stating our main result, we shall need the following hypotheses which restrict the behaviour of solutions as they cross the Ψ and Φ surfaces.

We shall assume:

(H_1) For all t_1 ∈ [0, t_0], (x_1, y_1, z_1) ∈ $I_E(S_1, 0, t_1) \cap S_{\Psi(t_1)}$ implies that there exists a solution of (1.1) emanating from (t, x_1, y_1, z_1) with trajectory which is on or above the Ψ surface, on some right neighbourhood of t_1.

(H_2) For all t_1 ∈ [0, t_0], (x_1, y_1, z_1) ∈ $I_E(S_1, 0, t_1) \cap S_{\Phi(t_1)}$ implies that there exists a solution of (1.1) emanating from (t, x_1, y_1, z_1) with trajectory which is on or below the Φ surface, on some right neighbourhood of t_1.

2. Main Results

Our first result describes qualitatively the behaviour of solutions of (1.1) assuming (**H_1**) and (**H_2**) holds.

Theorem 2.1

Let S_1 be a compact connected set in C (0) which intersects $S_{\Psi(0)}$ and $S_{\Phi(0)}$. Then

(i) $I_E(S_1, 0, t)$ contains a compact connected component in C(t) which intersects both $S_{\Psi(t)}$ and $S_{\Phi(t)}$ for all t_1 ∈ [0, t_0], or

(ii) There is a solution of (1.1) with (x(0), y(0), z(0)) ∈ S_1 having a maximal interval of [0, t^+] ⊂ [0, t_0] such that $|y(t)| \to \infty$, $|z(t)| \to \infty$ at $t \to t^+$.

Proof: Let E' = [(t, x, y, z)∈ E, Φ (t, y, z) – 1 <x <Ψ(t, y, z) + 1]. Assume there is no number K such that for all t∈ [0, t_0], 0, t) implies $|x| \le$ K, $|y| \le$ K, $|z| \le$ K. Then there is some interval [0, t^+] ⊂ [0, t_0] such that for any K > 0 there exists an $\varepsilon(K)$> 0 such that either |x|> K, or |y|> K, or |z|> K for t∈ [$t^+ - \varepsilon$, t^+].

Take any sequence $t_n \to t^+$ and let (x_n, y_n, z_n)∈ $I_{E'}(S_1, 0, t_n)$. Then either $|x_n| \to \infty$ or $|y_n| \to \infty$ or $|z_n| \to \infty$. By the continuity of Ψ(t, y, z) and Φ (t, y, z) it follows that if $|x_n| \to \infty$ then $|y_n| \to \infty$, $|z_n| \to \infty$. Hence either case we have $|y_n| \to \infty$, $|z_n| \to \infty$.

By a standard diagonalization process,using the solutions associated $\{(x_n, y_n, z_n)\}$, we may construct a solutions $(x(t), y(t), z(t))$of (1.1) with (x(0), y(0), z(0)) ∈ S_1 which exists on [0,t]⊂ [0, t^+] and such that $|y(t)| \to \infty$, $|z(t)| \to \infty$ as $t \to t^+$.

Moreover, (t, x(t), y(t), z(t))∈ E' ⊂ E. This is the case (ii), hence to complete the proof we may assume there exists a number K > 0, such that for all t ∈ (0, t_0],

(x, y, z)∈ $I_{E'}(S_1, 0, t)$ implies $|x| \le$ K, $|y| \le$ K, $|z| \le$ K.

Let T be the set of all points $\bar{t}$ ∈[0, t_0] such that for all t ∈[0, $\bar{t}$], the set $I_{E'}(S_1, 0, t)$ contains a component in C(t) which intersects both $S_{\Psi(t)}$ and $S_{\Phi(t)}$. Then T is nonempty since 0 ∈ T and is bounded above by t_0. We shall show t is closed and thus if we let

s = Sup T, then s = t_0 implies the conclusion of theorem (1.1).

Let $\{s_i\}$ be a sequence of points in T converging to s with $C_i \subset C(s_i)$ a component of $I_{E'}(S_1$, 0, $s_i)$ which intersects both $S_{\Psi(s_i)}$ and $S_{\Phi(s_i)}$. Let L denote the limit set of $\{C_i\}$ and let (a, b, c) ∈ L, since L is nonempty. Then there exists a sequence $\{(x_i', y_i', z_i')\}$ converging to (a, b, c) where $(x_i', y_i', z_i') \in C_i'$ and $\{C_i'\}$ is a subsequence of $\{C_i\}$.

Let L'be the set of limit points of $\{C_i'\}$; hence L' is compact and since C_i' intersects both $S_{\Psi(s_i')}$ and $S_{\Phi(s_i')}$ for all i then L'intersects both $S_{\Psi(s)}$ and $S_{\Phi(s)}$.

If L' contains no component which intersects both $S_{\Psi(s)}$ and $S_{\Phi(s)}$, then since L' is compact and intersects both $S_{\Psi(s)}$ and $S_{\Phi(s)}$, L' is the union of two non-empty compact sets M and N which are separated by an arc A such that A $\cap L' = \emptyset$.

Assume (a, b, c) ∈ M and (c, d, e) be any point in N and let $(\bar{x}_i'', \bar{y}_i'', \bar{z}_i'') \to$ (c, d, e) , where $(\bar{x}_i'', \bar{y}_i'', \bar{z}_i'') \in C_i''$ a subsequence of $\{C_i'\}$. Choose $\{(x_i'', y_i'', z_i'')\}$ to be the subsequence of $\{(x_i', y_i', z_i')\}$ contained in C_i''. For i sufficiently large the connected set $I_{E'}(C_i'', s_i''$,s) intersect A and let (p_i, q_i, r_i) be the points of intersection.

Thus $(p_i, q_i, r_i) \in I_{E'}(C_i'', s_i''$,s)$\cap$ S.

There exists a point (p, q, r) which is limit point of $\{(p_i, q_i, r_i)\}$ and since $s_i'' \to$ s, (p, q, r) is a limit point of $\{C_i''\}$. However then (p, q, r) $\in L'$ and (p, q, r) ∈ A$\cap L'$. This is a contradiction and we conclude L' and thus $I_{E'}(S_1$, 0, s) contains a component c in C(S) intersection both $S_{\Phi(s)}$ and $S_{\Psi(s)}$. Thus T is closed.

By assumptions (H_1) and (H_2) there exists a δ> 0 such that

$I_{E'}$(C, s, t)⊂[(x, y, z): (t, x, y, z)∈ E'] and $I_{E'}$(C, s, t) contains a component which intersects both $S_{\Phi(t)}$ and $S_{\Psi(t)}$ for all t ∈ (s, s +).

However this contradicts the fact that s = Sup T. Thus s = t_0 and (i) is proved.This completes the proof of theorem.

Theorem 2.1 with conditions that restrict the possibility of (ii) occurring can be used to deduce existence theorems.

The following condition will thus be imposed:

(H_3) Given any n > 0 and t_0> 0 there exists a number N(t_0, n) such that for any solution (x(t), y(t), z(t)) of (1.1) with $|y(0)| < n_1$, $|z(0)| < n_2$ and (t, x(t), y(t), z(t))∈ E for t ∈ [0, t_0) we have $|y(t)| <$ N(t_0, n_1), $|z(0)| <$ N(t_0, n_2), for all t ∈ [0, t_0).

Theorem 2.2 (Existence Theorem)

Assume conditions (H_1) - (H_3) hold. Let S_1 be a compact connected in C (0) which intersects both $S_{\Psi(0)}$ and $S_{\Phi(0)}$. Let S_2 be a closed connected subset of C (t_0) such that $S_2 \cap$ [(x,y) : y is arbitrary] $\neq \emptyset$, then there exists a solution of (x(t), y(t), z(t)) of (1.1) on [0, t_0] such that (x(0), y(0), z(0)) ∈ S_1, (x(t_0), y(t_0), z(t_0)) ∈ S_2 with (t, x (t), y (t), z (t)) ∈ E for t ∈ [0, t_0].

Proof: Since S_1 is a compact set, let n_1 = Sup$|y|$, n_2 = Sup$|z|$ for (x, y, z)$\in S_1$, and let N = N(t_0, n) be as in (H_3). Then$|y(t)|$<N(t_0, n_1) for any (x(t), y(t), z(t)) of (1.1) with (x(0), y(0), z(0))$\in S_1$ and all t $\in$ [0, t_0]. By theorem 2.1, $I_E(S_1$, 0, t_0) contains a compact connected component c to C(t_0) which intersects both $S_{\Psi(t_0)}$ and $S_{\Phi(t_0)}$. The conditions imposed on S_2 insure that $S_2 \cap I_E(S_1$, 0, $t_0) \neq \emptyset$. This concludes the proof of the theorem.

Another approach to this problem is the application of Wazewski's topological method.
Consider the differential system

$$\left.\begin{aligned} x' &= f(t,x,y), \quad x(t_0) = x_0, \quad t_0 > 0 \\ y' &= f(t,x,y), \quad y(t_0) = y_0, \quad t_0 \geq 0 \end{aligned}\right\} \text{--- (2.1)}$$

Where f $\in$ C [Ω, R^n], Ω being any open set in R^{n+1}. Let Ω_0 be an open set of Ω, $\partial\Omega_0$ is the boundary and $\overline{\Omega_0}$ the closure of Ω_0.

Definition 2.1

A point $(t_0, x_0) \in \Omega \cap \partial\Omega_0$ and $(t_0, y_0) \in \Omega \cap \partial\Omega_0$ is said to be egress point of Ω_0 with respect to the system (2.1)if, for every solution x(t), y(t) of (2.1), there is an ϵ> 0 such that (t, x(t)) $\in \Omega_0$, (t, y(t)) $\in \Omega_0$ for $t_0 - \epsilon \leq$ t <t_0.

An egress point (t_0, x_0) and (t_0, y_0) of Ω_0 is called a strict egress point of Ω_0,
if (t, x (t)) $\notin \overline{\Omega_0}$ and (t, y (t)) $\notin \overline{\Omega_0}$ for t_0< t $\leq t_0 + \epsilon$ for a ϵ> 0.

Denote the set of all points of egress (strict egress) as $\mathbb{S}$ ($\mathbb{S}^*$).

Definition 2.2

If A $\subset$ B are any two sets of a topological space X and π : B $\rightarrow$ A is a continuous mapping from B into A such that $\pi(p) = p$ for every p $\in$ A, then π is said to be a retraction of B onto A.

When there exists a retraction of B onto A, A is called retract of B.

Theorem 2.3

Let f $\in$ C (Ω, R^n), Ω open in R^{n+1}. Assume that through every point of Ω there passes a unique solution of (2.1). Let Ω_0 be an open subset of Ω. Suppose that all egress points of Ω_0 are strict egress points. Let U be a nonempty subset of $\Omega_0 \cup \mathbb{S}$ such that U $\cap \mathbb{S}$ is a retract of $\mathbb{S}$, but not a retract of U. Then there exists at least one point $(t_0, x_0, y_0)\in$ U $\cap\Omega_0$ such that the solution(t, x(t), y(t))of (2.1) remains in Ω_0 on its maximal interval of existence to the right of t_0.

We now apply theorem 2.3 to BVP prescribed in theorem 2.2. Although we assume uniqueness of solutions of (1.1) this is not essential as a Wazewski-like theorem for non-uniqueness has been developed. Let Ω = [(t, x, y, z): t$\geq$ 0] - S_2; then Ω is a relatively the half space open subset of [(t, x, y, z): t$\geq$ 0].

Let Ω_0 = [(t, x, y, z) : 0 $\leq$ t<t_1,Φ(t) < x <Ψ(t), $|y|<\infty$, $|z|<\infty$] where now Ψ(t, y, z), Φ(t, y, z) are independent of y and z. Let U = S_1.

From hypothesis (H_1) and (H_2), it is not difficult to see that $\mathbb{S} = \mathbb{S}^*$, and $\mathbb{S}$ consists of the union of the sets